Justifications, ECQ and relevancy: *First comments*

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08.09.2019

Overview [1]

Objective: We analyze ECQ by taking into account some relevant reasons for rejecting it. Then I suggest why and how some of these positions can be related to Justification Logic and in particular it's Application Operation.

[1] Special thanks to the Consejo Nacional de Ciencia y Tecnología, Conacyt, which has supported this research with a PhD grant

Overview [2]

Objective: Specifically, I try to show that a particular criticism in relevance logic (Restall, Dunn, 2002) against ECQ has important ressemblances wrt. to syntactic rules in Basic Justification Logic. We state a new problem: how can we define notions to compare both logics? In a similar vein, I conjecture that an answer to this problem is in the vecinity of Fitting's approach (2017).

[1]Special thanks to the Consejo Nacional de Ciencia y Tecnología, Conacyt, which has supported this research with a PhD grant

1. A quick reminder of ECQ
2. Relevant reasons against *bad* inferences
3. Justification Logic Approach and relevance

A quick reminder of ECQ

- Objectives

Ex contradictione quodlibet (ECQ) comes from an argument of CPL (classical) which is taken as validating that from a contradiction everything follows, i.e. if $\{A, \neg A\}$ then Q, for any inference.

- Scope and interest.

A quick reminder of ECQ

Ex contradictione quodlibet (ECQ):

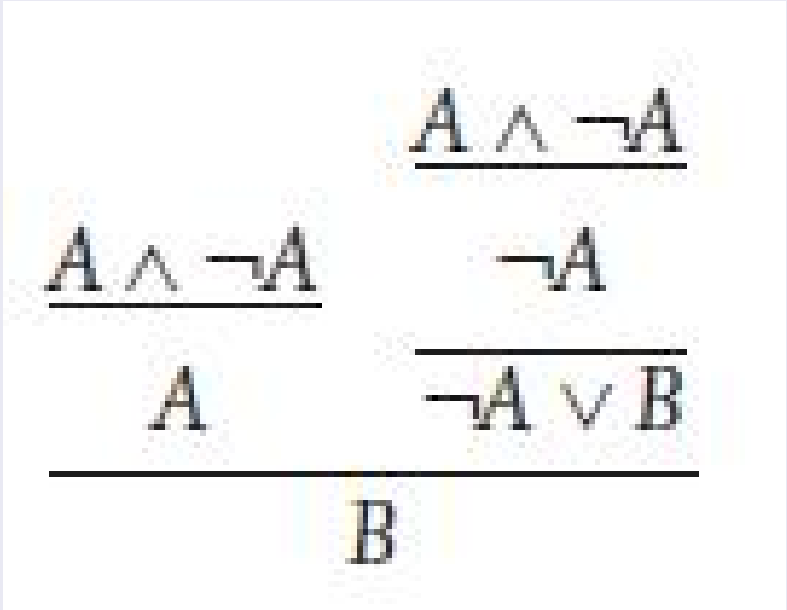
Principle of Explosion or ex contradictione quodlibet (*ECQ*)

1. $A \wedge \neg A$	*Ass.*
2. A	$E \wedge -r; 1$
3. $\neg A$	$E \wedge -l; 1$
4. $A \vee B$	$I \vee -r; 2$
5. B	$DS \vee -r, 4, 3$ $\square$

A quick reminder of ECQ

Principle of Explosion or ex contradictione quodlibet (*ECQ*)

- Structure of the argument! (Priest, 2008, 4.9.2)



We restrict our attention only for this form of the ECQ.

Relevant reasons against *bad* inferences

- A rejection of ECQ can be given in terms of the "lack of [*real*] connection between the premisses and conclusion"(Mares, 2012), *cfr.* (Anderson, Belnap, 1975, ... a lot)
- NO: The notion of relevant connection in a non-logical way related to topic-relevant-content between Premisses and Conclusion is not of our interest.
- For the moment, we will leave aside of this discussion the answer for the intentional stance of $\vee$... (until the last part.).

Relevant reasons against *bad* inferences

- Point of view: some properties or principles of relevance have been given to analyze the notion of *relevance* in logical settings.
- So, we restate: Can the rejection of ECQ be given in terms of the "lack of [*real*] connection between the premisses and conclusion"through relevant constraints or principles? (SEP, Relevant logic)). (Anderson, Belnap, 1975, ... a lot)
- Let's see: some properties or principles of relevance have been given, for ex. Variable Sharing principle (VSP) (others... interesting but not for today!). It's a necessary non-sufficient condition.
- The inference should not be material conditional, but also not the strict conditional, because of other reasons -the paradoxes-...
- People has worked in proof theory, structural (substructural) theories and/or semantics to give a relevantist account.

Relevant reasons against *bad* inferences

Perhaps, the Variable Sharing Principle (VSP) can reject the ECQ inference? ...

VSP:

No formula of the form $A \rightarrow B$ can be proven in a relevance logic if A and B do not have at least one propositional variable (sometimes called a proposition letter) in common and that no inference can be shown valid if the premises and conclusion do not share at least one propositional variable. cfr.(Mares, 2012)

This one seems to be semantically undetermined/neutral.

Relevant reasons against *bad* inferences

But take this into account, for example:

> If A $\rightarrow$ B is a theorem of R (or E), then there exists some sentential variable p that occurs in both A and B. This is understood by Anderson and Belnap as requiring some commonality of meaning between antecedent and consequent of logically true relevant implications. The proof uses an ingenious logical matrix, having eight values, for which see [Anderson and Belnap, 1975, Section 22.1.3]... (Restall, Dunn, 1.6, p. 28)

This is semantically dependent/not neutral.

Relevant reasons against *bad* inferences

- After those consideration, does the Lewis argument fails prey of (VSP)?
  I think that the structure of ECQ shows that we can't answer this so easily.
- All the inferences of the Lewis independent argument for ECQ are ok with the VSP just by looking it step by step.

Relevant reasons against *bad* inferences

This seems to be an error. Why? Let's see only the more syntactic notions and leave for other place the more semantically loaded discussion

- But consider that most of the approaches do a semantic development for this: (Fine, 1974), (Routley, Routley, 1972), (Routley-Meyer, 1993). Also algebraic approach (Dunn, Restall, 2002:98ff). Lots of works by Carnielli. We don't consider algebraic or set theoretic notion also... :(

Relevant reasons against *bad* inferences

We take just the next argument to engage in our discussion:
Restall-Dunn has argued that arguments the indep. arg. just commits some errors:

> *Remember that the Introduction rules provide the actual inferences, perhaps later to be released again as actual inferences by elimination rules. The problem with the disjunctive syllogism is that it can release inferences from $\vee$ that it just does not contain. In another context, [Anderson, Belnap, 1962, 1975] observed that Gentzen-style rules for a given connective should be 'conservative', i.e. they should not create new inferences not involving the given connective.).* (Relevance Logic, Dunn Restall, p. 35)

(Ref. Citation A)

Relevant reasons against *bad* inferences

In fact the authors continue with this explanation qualyfing as the culprit the rule of Disjunctive Syllogism and diagnosing that the Lewis argument is viciously circular because it requires to in fact validate what it is at stake.

Thus the problem with the disjunctive syllogism is just that $p \vee q$ might have been introduced into the discourse (as it is in the Lewis 'proof') by $\vee$-Introduction from p. So then to go on to infer q from '$p \vee q$' and $\neg p$ by the disjunctive syllogism would be legitimate only if the inference from p, $\neg p$ to q were legitimate. But this is precisely the point at issue. At the very least the Lewis argument is circular (and not independent). (Relevance Logic, Dunn Restall, p. 35)

(Ref. Citation B)

Consequences of the analysis

The previous argument involves as a consequence, like some logicians has clarified, that SD "is just Modus ponens for the material conditional" (Dunn, Restall, 2002) (see gama-rule clarifications), (Priest, 8.6.5).

- Nevertheless, the object language lacks the explanation about why this must be. Usually the answers are provided by means of Semantic considerations (or intentions)
  These considerations are prone to a lot of argument.

Relevant reasons against *bad* inferences

- Is it really a problem?
- Consider D. Lewis position against relevantist Routley and Priest interpreted in more ontological-metaphysical sense, so to speak:

"The reason we should reject this proposal is simple. No truth does have, and no truth could have, a true negation [discussing a position which accepts departure from real"bivalences]. Nothing is, and nothing could be, literally both true and false. This we know for certain, and a priori, and without any exception for especially perplexing subject matters. The radical case for relevance should be dismissed just because the hypothesis it requires us to entertain is inconsistent" (Lewis, Log. for equivocators, p. 434)

What to do?

- Of course, even if by themeselves, semantic approaches are robust in a logical sense, it would be worth if there's a different more syntactic way of proceding (apart from the already existent ones).
- Following a similar reasoning, it's interesting to see if there's a way of explaining what's happening in the ECQ just by means of adding some simple syntactic elements an, at the same time respecting the (Restall, Dunn 2002) criticism.

JL, and Cases of different derivations

- Here I will treat the issue by means of the hypothetical proofs which are postulated. This is a powerful and clarificatory way of seeing what is at stake in the argument.
- It's important that the real use of premisses must be taken into account also.
- But it seems very reasonable to have some kind of notion of inconsistent information (corpus of data) from which we can reason logically, something which is the case also by D. Lewis (Log. for equivocators, p. 437) (In fact, he rejects relevance on the implication in this sense but because of other problems of it's inferences)

Justification Logic. Traditional approach, quick reminder

- Syntax

Let CS be an infinite set of constant justification terms, and let V be a different infinite set of variable justification terms. Let t, u be a any kind of justification term.

$$t, u ::= c \mid x \mid (t * u) \mid (t + u) \mid (!t) \mid (?t);$$

s.t. $c \in C$ and $x \in V$. If $t, u \in C$ or $t, u \in V$, t is an atomic term.

Justification Logic. Traditional approach

- Syntax

Language of *JL* (*wff*)

$$\varphi, \psi ::= p \mid \neg\varphi \mid \varphi \vee \psi \mid t : \varphi;$$

where $p \in Atm$ and $t \in Tm$
We can obtain the induced *wff* for the standard connectives, i.e. (for notational purposes), $\varphi \to \psi := \neg\varphi \vee \psi$; $\varphi \wedge \psi := \neg(\neg\varphi \vee \neg\psi)$; $\varphi \leftrightarrow \psi := (\varphi \to \psi) \wedge (\psi \to \varphi)$.

Justification Logic, syntax

- Axiomatization.

Let φ and ψ be formulas; also, suppose t and s are terms.

1. • A1. TAUT formulas, where $TAUT = \{\varphi \mid \varphi \textit{ is propositionally valid}\}$
2. • A2. Application Axiom: $s : (\varphi \to \psi) \to (t : \varphi \to (s * t) : \psi)$
3. • A3. Sum Axiom: $(s : \varphi \to (s + t) : \varphi), (s : \varphi \to (t + s) : \varphi)$

- Rules.

1. • $R1$. Modus Ponens: $\varphi \to \psi$ and $\varphi \Rightarrow \psi$
2. • $R2$. Constant Specification: $\Rightarrow c : A$, where $c \in CS$ and A is an axiom.
3. • $R2!$. Constant Specification with positive introspection $\Rightarrow !...!c : c : A$, for any A.

The previous system forms the Basic Justification Logic which is denoted with $J - \emptyset$.

Justification Logic

- It's important to notice that we can add some other axioms to the basic $J - \emptyset$ system.
  In JL, we add some axioms in a similar vein wrt. Modal Logic (Blackburn, 2002) but the semantic behavior of the system must be handle in very specific ways. Here some examples (Artemov, 2008) (Studer, 2012)

1. • A4. Factivity Axiom: $t : \varphi \rightarrow \varphi$
2. • A5. Positive Introspection Axiom: $t : \varphi \rightarrow !t : t\varphi$
3. • A6. Negative Introspection Axiom: $\neg t : \varphi \rightarrow ?t : \neg t : \varphi$
4. • A7. Coherence (No conflicts) Axiom: $t : \bot \rightarrow \bot$

Justification Logic

- *CS* can handle different shades of justified formulas Let *CS* be an arbitrary set of*JL-wff* of the form $c_i : \phi_i$, s. t. $\phi_i \in TAUT$ and $c_i \in C$.
  It is the case that:
  $J - CS =$ def $J - \emptyset + CS$.
  $J - CS$ is any Justification system with additional axioms, where these axioms are elements of a *CS*.
  Some property a *CS* can have are the following:
  • *Axiomatically appropriate* $CS = \{c_i : \varphi_i |\ c_i \in C \text{ and } \varphi_i \in TAUT\}$.
  There are more...

Justification Logic

- Semantics.
- Traditional version
- Mkrytchev, Kripke-Fitting, Modular models, Subset models...

Justification Logic

- Semantics, Basic Modular Models.
- Definition:

Let $L(X) = LJ \cup X$, where $LJ = \{A1 - A3\}$, i.e. the set of basic JL and let $X \subseteq \{A4 - A7\}$, i.e. the set of non basic Axioms of JL-systems. We let CS to be as in $J - CS$.

Justification Logic

- Semantics.
- Basic Modular Models:

Modular models

Let $X, Y \subseteq LJ$ and $t \in Tm$, and define:

$(I)\, X * Y := \{\varphi \in LJ \mid \psi \to \varphi \in X \text{ and } \psi \in Y \text{ for some } Y \in LJ\}$

$(II)\, t : X := \{t : \varphi \mid \varphi \in X\}$

...

Justification Logic

- Semantics.
- Basic Modular Models:

We introduce now the basic evaluation in modular models:

Evaluation

$\star : PROP \mapsto \{1, 0\}$, and
$\star : Tm \mapsto \mathcal{P}(LJ)$,
s.t. for arbitrary justifications $s, t \in Tm$ and for every φ,
$s^\star * s^\star \subseteq (s * t)^\star$;
$s^\star \cup t^\star \subseteq (s + t)^\star$;
$\varphi \in s^\star$ for any conclusion $t : \varphi$ of LJ or $L(X)$ if $(t, \varphi) \in CS$ is being adopted, where $s^\star$, $s \in Tm$ denotes $\star(s)$. This notation can be used also for atoms, i.e. $p^\star$ is $\star(p)$.
Following these lines, the satisfaction under evaluation $\star$ is defined inductively in a direct way.

Justification Logic

- Lots of systems. For every (normal) modal logic there's a counterpart of Justification Logic.
- Realization theorem, Forgetful projection, etc.
- Approach: *Tableaux* Ghari (2016)

JL, and Cases of different derivations

- The diagnosis is that ECQ is explained by means of Justification Logic, *JL*, in particular by interpreting hypothetical proofs as proofs with variables.

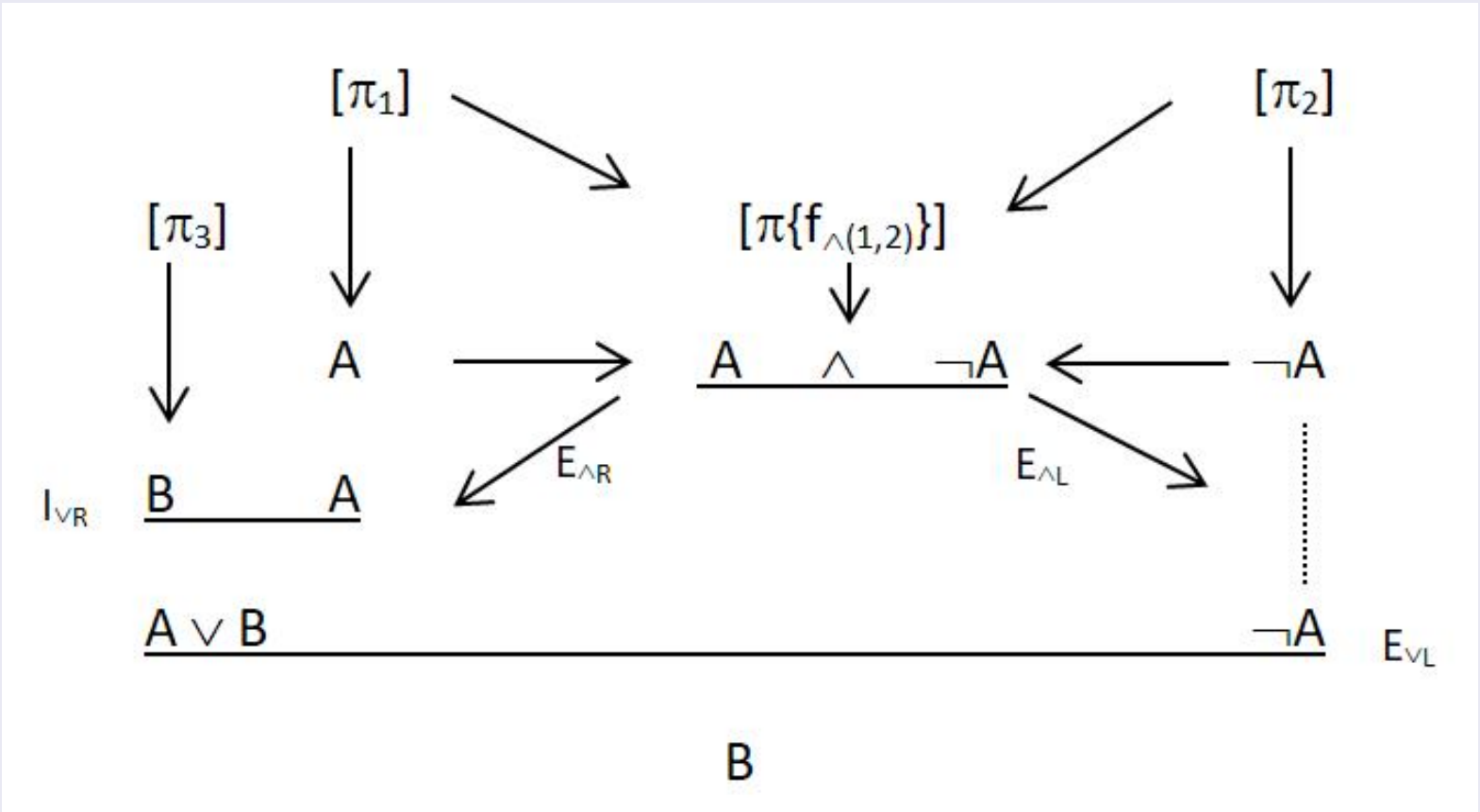
title

The use of types of Justification Logic *JL* shows in a more object language (syntactic) sense what's happening in the argument of ECQ, how and why, by means of its tracking properties of the inferences. ...

JL, and Cases of different derivations

- General analysis

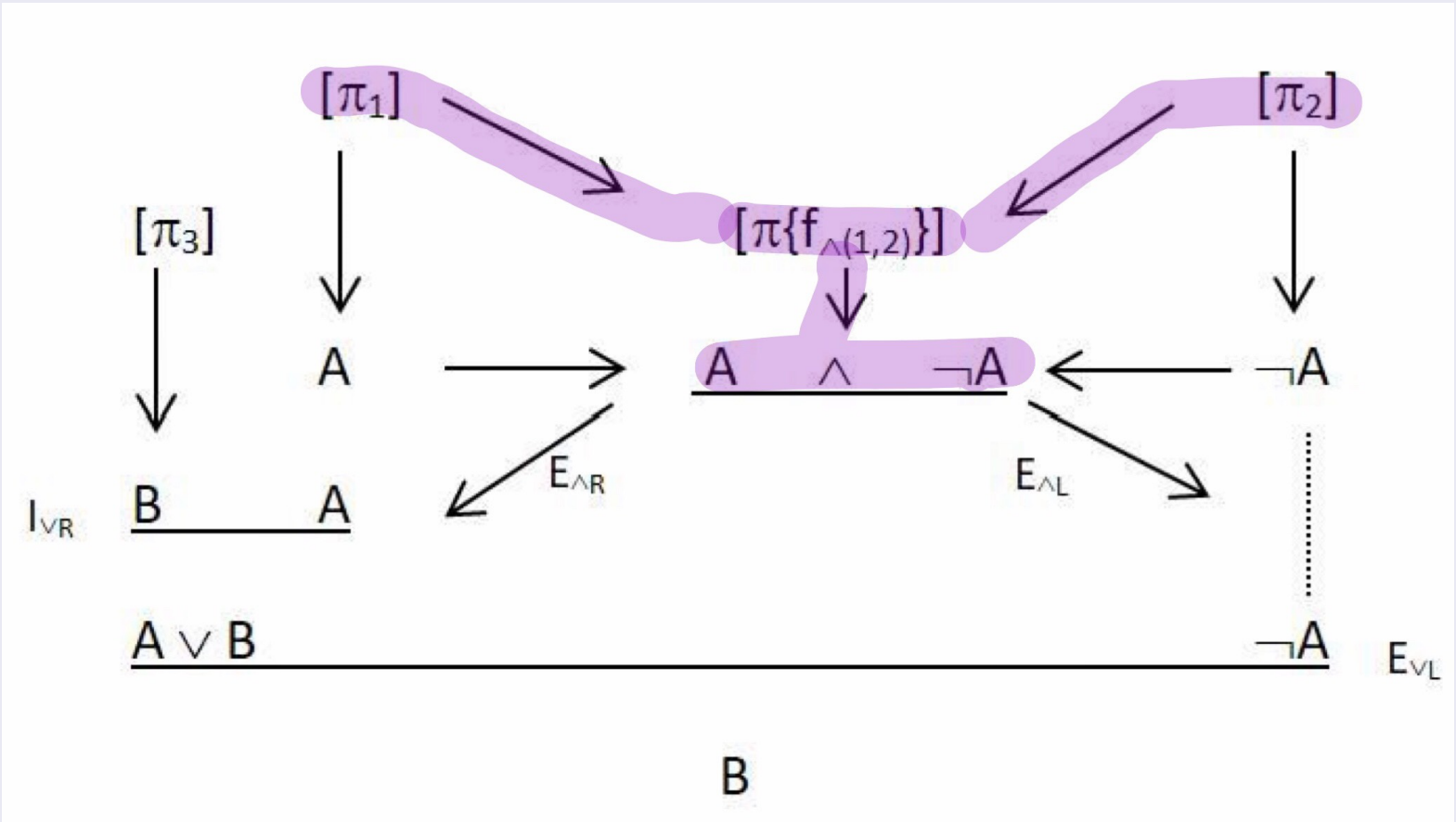
The general structure of JL-EQL



JL, and Cases of different derivations

- Observation 1

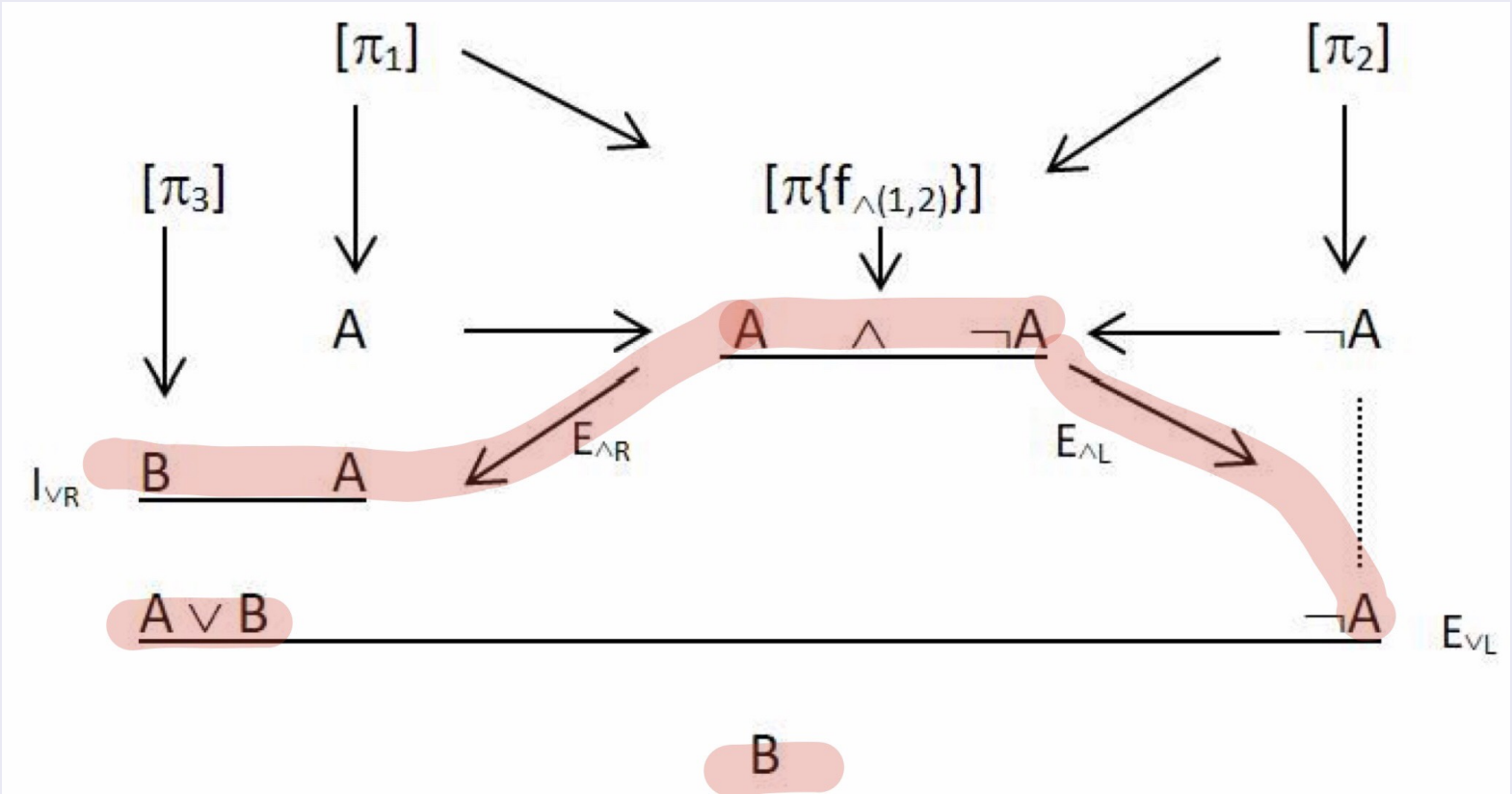
Case 1



JL, and Cases of different derivations

- Observation 2

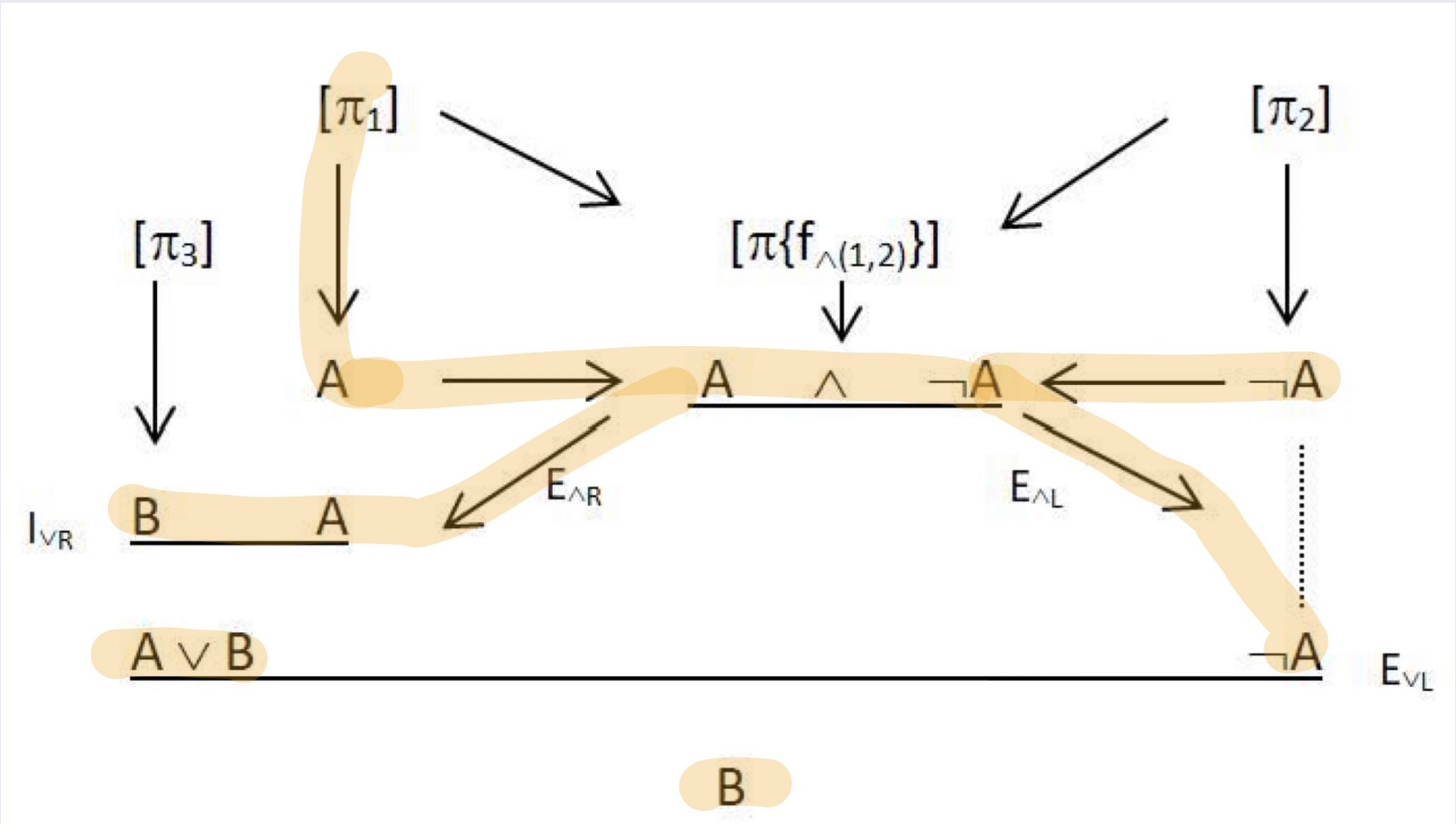
Case 2



JL, and Cases of different derivations

- Observation 3

Case 3



JL, and Cases of different derivations

Then ECQ remains in TAU, i.e. when we don't have justifications, but it doesn't in general when we have justifications, at least when we can have combinations:

- non-justified formulas
- justified formulas w. CS
- justified formulas w. Var.
- justified formulas w. Var and CS ...

Why this option?

- It's interesting to clarify by means of JL or other formal justification-evidentiary analysis over conflicting operations on justification polynomial to what extent notions of relevance can be treated
- Interestingly, it seems that this has something to do with how justification is related to paraconsistent (or FDE-evidentiary justification systems) and paracomplete issues.
- Moreover, it seems to be a a dispensable issue the presence of gluts and gaps in the new models (we respect the old modularity models of JL)

Why this option?

Why this other option? Because it is promising!!!

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Because it can provide more developments under the non-classical + justification logic stream.

- Notice that it seems very promising to compare approaches such as (Fitting, 2017) -evidence in favor and evidence against, or (Studer, Lehmann, 2017) with the insights of semantic constraints such as the ones in paraconsistent and paracomplete logics, but at the level of justification terms.

Why this option?

Why this other option? Because it is promising!!!

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Because it can provide more developments under the non-classical + justification logic stream.

- Another example: the Routley-Meyer ternary relation interpreted in PWS through Dunn's informational notion ressembles some kind of interesting relations with justification application.
- Of course, it's all an invitation to work on these matters!!!

Děkuji!