Squares and Uncountably Singularized Cardinals

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Joint work with Dima Sinapova!

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Theorem (Shelah)

If $\aleph_\omega$ *is a strong limit (in particular if* GCH *holds below* $\aleph_\omega$*) then* $2^{\aleph_\omega} < \aleph_{\omega_4}$.

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But what actually *happens* when we singularize cardinals?

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- When we speak of a "square sequence" we are often declining to specify λ for $\square_{\kappa,\lambda}$.

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- $\square^*_\kappa$ is equivalent to a special κ^+-Aronszajn tree.
- If $\mu < \kappa$ are infinite and $\square_\kappa$ holds, then $(\kappa^+, \kappa) \twoheadrightarrow (\mu^+, \mu)$ fails.

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Fact (Gitik-Sharon)

Assuming large cardinals, there are models $\mathrm{ON} \subset V \subset W$ and a cardinal κ such that:

- $V \models$ "*κ is inaccessible*",
- $(\mathrm{cf}\,\kappa)^W = \omega$,
- *and $\square^*_\kappa$ fails in W (hence $\square_{\kappa,\omega}$ fails in W).*

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It is known that if μ is a Mahlo cardinal and $\mathrm{Col}(\kappa, <\mu)$ is the Lévy Collapse for making μ into κ^+, then $\square_{\kappa,\tau}$ fails in $V[\mathrm{Col}(\kappa, <\mu)]$ for all $\tau < \kappa$.

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- In $\bar{V}[\mathrm{Col}(\kappa, <\mu)]$, let $\mathbb{M}$ be Magidor's variation of Prikry forcing for singularizing κ to have an uncountable cofinality λ.

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- Then the statement of the theorem holds if $V = \bar{V}[\mathrm{Col}(\kappa, <\mu)]$ and $W = \bar{V}[\mathrm{Col}(\kappa, <\mu) * \mathbb{M}]$.

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The most important technical ingredient is the Prikry Density Lemma for Magidor Forcing.

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Is there necessarily a $\square_{\kappa,<\kappa}$*-sequence?*

Děkuji!