

Logic and Natural Language

Commitments and Constraints

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Outline

- 1 Introduction
- 2 Logic and Natural Language
 - The Linguistic Project
 - The Traditional Project
- 3 Formalization
- 4 Semantic Constraints as Commitments
 - Semantic Constraints (2014)
 - Formalization Constraints (2019)

Natural Language

Formal Language

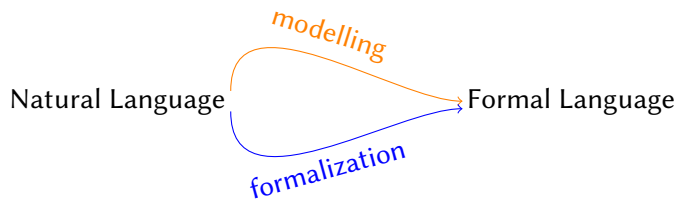
Natural Language

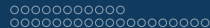
(Expression of Content)

Formal Language

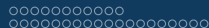
(Logic)







Logic in Linguistics



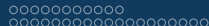
Logic in Linguistics

The tools of formal logic are used in the descriptive study of natural language (Montague, Davidson, Montague, Chomsky).



Logic as a Methodology for the Sciences

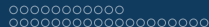
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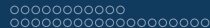
Its first purpose, therefore, is to provide us with the most reliable test of the validity of a chain of inferences and to point out every presupposition that tries to sneak in unnoticed (Frege 1879, p. 6)



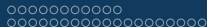
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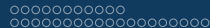


Normative Matters



Normative Matters

- *logical norms might provide first-personal **directives** that guide the reasoner;*
 - *logical norms might serve to make third-personal **evaluations**, setting standards or ideals by which to assess an agent's doxastic state for its logical cogency; or, finally,*
 - *logical norms might play the role of third-personal **appraisals** by which we criticize, blame, or otherwise hold accountable an agent for her doxastic conduct.*
- (Steinberger 2019, p. 7)*



Argument Reconstruction (Brun 2014)

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This process includes a stage of argument analysis, producing informal inferences where the premisses and conclusions are spelled out, followed by formalization, which produces formal arguments which can be assessed by the rules of a formal system.

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It is certainly possible to recognize from its form alone that a sentence is analytic; but only if the syntactical rules of the language are given. (Carnap 1937, p. 186)

Why formalize?

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For let the first terms, of the combination of which all others consist, be designated by signs; these signs will be a kind of alphabet. It will be convenient for the signs to be as natural as possible – e.g., for one, a point; for numbers, points; for the relations of one entity with another, lines; for the variation of angles and of extremities in lines, kinds of relations. If these are correctly and ingeniously established, this universal writing will be as easy as it is common, and will be capable of being read without any dictionary; at the same time, a fundamental knowledge of all things will be obtained. . . .

(Leibniz 1966, 10–11)

Why formalize?

*. . . The whole of such a writing will be made of geometrical figures, as it were, and of a kind of pictures – just as the ancient Egyptians did, and the Chinese do today. Their pictures, however, are not reduced to a fixed alphabet . . . with the result that a tremendous strain on the memory is necessary, which is the contrary of what we propose.
(Leibniz 1966, 10–11)*

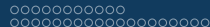
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Formalization provides the ground for logical norms to be applied.



Adequate Formalization

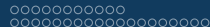
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Yes: Resnik (1985), Peregrin and Svoboda (2013, 2017), and Brun (2014, 2017)

No: Thagard (1982) and Baumgartner and Lampert (2008)

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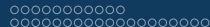
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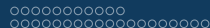
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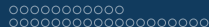
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- **Empirical linguistics/psychology have a decisive role?**

Yes: Thagard (1982)

No: Resnik (1985)



Adequate Formalization

[L]ogical forms are not simply given and are not found by sheer abstraction. . . [F]ormalizing is not merely abstracting but also involves creative and normative aspects of constructing logical forms (Brun 2014)

Explication (Carnap 1962)

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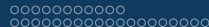
The rules of use of the explicatum are given explicitly, and are constitutive of it.

Explicitly Stated Rules

[T]he rules which are presented in scholarly books as the rules of logic—we could speak about logical rules in the narrow sense—are in our view not something merely discovered or brought to light by philosophers or logicians, but rather something that acquired a definite shape only after it was explicitly articulated within a theory. (Peregrin & Svoboda 2017, p. 10-11)

Summary up till now

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Summary up till now

- The explicitly stated rules of a formal language are constitutive of it, and their status as such is the source of their normative force.
- When employing a formal language, one is subject to evaluation vis-à-vis its rules. Indeed, when one uses a formal language, one thereby commits oneself to the norms of following its rules.
- It is important that the rules are explicit, because only then can one properly commit to them and be evaluated according to them. Natural language is a complex, dynamic, natural phenomenon. When we formalize, we rigidify language (see also Peregrin & Svoboda 2017, p. 3).



Semantic Constraints

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$$(\wedge): I(\varphi \wedge \psi) = T \Leftrightarrow I(\varphi) = T \text{ and } I(\psi) = T$$

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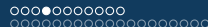
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allRed, *allGreen*

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$I(\text{allRed})$, $I(\text{allGreen})$



Semantic Constraints

Fixing something amounts to limiting the admissible interpretations.

$$I(\textit{allRed}) \cap I(\textit{allGreen}) = \emptyset$$



- $I(\text{even}) \cap I(\text{odd}) = \emptyset$
- $I(\text{bachelor}) \subseteq I(\text{unmarried})$
- $I(\text{H}_2\text{O}) = I(\text{water})$
- $I(\text{wasBought}) = I(\text{wasSold})$
- $I(\exists) = \{A \subseteq D : A \neq \emptyset\}$
- $I(\forall) \in \{\{B \subseteq D : A \subseteq B\} : A \subseteq D\}$
- $I(\text{most}) = \{\langle A, B \rangle \in \mathcal{P}(D)^2 : |A \cap B| > |A \setminus B|\}$
- $I(R)$ is a symmetric binary relation.
- $0 \in I(\text{naturalNumber})$
- $I(\text{prime}) = \{2, 3, 5, \dots\}$
- $|I(\text{Red})| = 375$ (i.e., the size of the extension of *Red* is 375.)

- $I(P) \subseteq D$
- $I(\text{John}) \in D$
- $I(\text{abc}) = T$ or $I(\text{abc}) = F$
- $I(d) \neq I(\wedge)$
- $I(\text{or}) \in \{f_{\vee}, f_{\underline{\vee}}\}$ where f_{\vee} is the inclusive or function, and $f_{\underline{\vee}}$ is the xor function from pairs of truth values to truth values.

The Language and its Models

Language

- Primitive expressions (*terms*)
- Complex expressions (*phrases*)

Models

$$M = \langle D, I \rangle$$

- D (the domain) is a non-empty set.
- I (the interpretation function) assigns to phrases values from the set-theoretic hierarchy with the members of $D \cup \{T, F\}$ as ur-elements.

Semantic Constraints

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Let Δ be a set of semantic constraints. A Δ -*model* is an *admissible model* by Δ , i.e. a model abiding by the constraints in Δ .

Sentences

A phrase p in L is a *sentence* (w.r.t. Δ) if for every Δ -model $M = \langle D, I \rangle$,

$$I(p) \in \{T, F\}$$

Logical Consequence

Let Δ be a set of constraints.

An argument $\langle \Gamma, \varphi \rangle$ is *Δ -valid* ($\Gamma \models_{\Delta} \varphi$) if for every Δ -model M , if all the sentences in Γ are true in M , then φ is true in M .



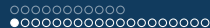
Semantic Meta-Constraints

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- Compositionality

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- Compositionality
- Closure under isomorphisms



Formalization Functions

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Source language (SL): primitive expressions and complex expressions
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Target language (TL): terms and phrases governed by semantic
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Source language (SL): primitive expressions and complex expressions (incl. declarative sentences)

Target language (TL): terms and phrases governed by semantic constraints

A *formalization function* F is a (possibly partial) function from a text in a source language to a target language, such that the domain of F is a subset of

$$\{ \langle \langle e_1, c_1 \rangle, \dots, \langle e_n, c_n \rangle \rangle :$$
$$n \in \mathbb{N}, \text{ for each } 1 \leq i \leq n, e_i \in SL, c_i \text{ is a context} \}$$

and the range is the set of phrases of TL.

it_{c₁} is_{c₂} raining_{c₃} and_{c₄} it_{c₅} is_{c₆} snowing_{c₇}

it_{c₈} is_{c₉} raining_{c₁₀}



$$\frac{p \wedge q}{p}$$

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A set of formalization constraints Π *determines the validity* of an argument in a given SL text if all Π -formalization functions agree on its validity.

$it_{c_1} is_{c_2} raining_{c_3} and_{c_4} it_{c_5} is_{c_6} snowing_{c_7}$

$it_{c_8} is_{c_9} raining_{c_{10}}$

$$\textcircled{1} F(\textit{and}_{c_4}) = \wedge$$

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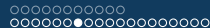
$$\textcircled{2} F(\textit{it}_{c_1} \textit{is}_{c_2} \textit{raining}_{c_3}) = F(\textit{it}_{c_8} \textit{is}_{c_9} \textit{raining}_{c_{10}})$$



- 1 $F(\mathit{and}_{c_4}) = \wedge$
- 2 $F(\mathit{it}_{c_1} \mathit{is}_{c_2} \mathit{raining}_{c_3}) = F(\mathit{it}_{c_8} \mathit{is}_{c_9} \mathit{raining}_{c_{10}})$
- 3 $F(\mathit{it}_{c_1} \mathit{is}_{c_2} \mathit{raining}_{c_3}) = p$
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$$this_{c_1} is_{c_2} red_{c_3}$$

$$this_{c_4} is_{c_5} red_{c_6}$$

$$\textcircled{1} F(\text{red}_{c_3}) = \text{allRed}$$

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$$\textcircled{2} F(\text{red}_{c_6}) = \text{allRed}$$

$$\textcircled{3} F(\text{this}_{c_1}) = a$$

$$\textcircled{4} F(\text{this}_{c_4}) = b$$


$$allRed(a)$$

$$allRed(b)$$

*this*_{c₁} *is*_{c₂} *red*_{c₃}

*this*_{c₄} *is*_{c₅} *not*_{c₆} *green*_{c₇}


$$allRed(a)$$

$$\neg allGreen(b)$$



Formalization Meta-Constraints

- *Sentencehood*. If e is a declarative sentence in SL, c a context, and Δ the set of TL semantic constraints then $F(e_c)$ is a sentence w.r.t Δ .

Formalization Meta-Constraints

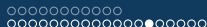
- *Sentencehood*. If e is a declarative sentence in SL, c a context, and Δ the set of TL semantic constraints then $F(e_c)$ is a sentence w.r.t Δ .
- *Correspondence*. The formalization of an argument should be valid iff the source argument is informally valid.

Formalization Meta-Constraints

The Principle of Univocality:

- (PU) For any expression e in the source language and any contexts c, c' ,

$$F(e_c) = F(e_{c'})$$



Formalization Meta-Constraints

Equivocation and Co-Reference:

- (I) If two occurrences of expressions in the source language denote distinct objects, they should be assigned different target expressions by the formalization function.
- (II) If two occurrences of expressions in the source language denote the same object, they should be assigned the same target expression by the formalization function.

(Iacona 2018, 71)



Formalization Meta-Constraints

- Assume $e \neq e'$, $c \neq c'$, e_c and $e'_{c'}$ denote distinct objects, then

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- $I(F(e_c)) = I(F(e'_{c'}))$

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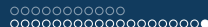


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- Formalization entails the explicit statement of rules and thereby to commitments on behalf of the one formalizing to the logical norms that may be associated with these rules.
- The framework of semantic constraints provides a formal backdrop for these commitments to be spelled out. We have seen how various principles regarding formalization can be expressed by semantic or formalization constraints or meta-constraints in this framework.



Thank You!