Monotonous and strong monotonous properties of some propositional proof systems for Classical and Non Classical Logics

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- In the theory of proof complexity main characteristics of the proof are: *t-complexity (length),* defined as the number of proof steps, *l-complexity (size),* defined as total number of proof symbols
- Let Φ be a proof system of any logic and φ be a tautology in this logic. We denote by t_{φ}^{Φ} (l_{φ}^{Φ}) the minimal possible value of $t - complexity$ $(l - complexity)$ for all proofs of tautology φ in Φ.

Definitions

- For every minimal tautology φ of fixed logic by S(φ) is denoted the set of all tautologies, which are results of a substitution in φ.

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- The proof system Φ is called ***t-strong monotonous (l-strong monotonous)***, if for every minimal tautology φ of this system and for every formula ψ from S(φ) $t_{\varphi}^{\Phi} \leq t_{\psi}^{\Phi}$ ($l_{\varphi}^{\Phi} \leq l_{\psi}^{\Phi}$)

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- The system ***GS*** based on generalization of splitting method

Definitions

- Following the usual terminology we call the variables and negated variables literals for classical logic. The conjunct K (clause) can be represented simply as a set of literals (no conjunct contains a variable and its negation simultaneously).

Definitions

- Each of the under-mentioned trivial identities for a propositional formula ψ is called *replacement-rule:*

$$0\&\psi = 0, \quad \psi\&0 = 0, \quad 1\&\psi = \psi, \quad \psi\&1 = \psi,$$

$$0 \vee \psi = \psi, \quad \psi \vee 0 = \psi, \quad 1 \vee \psi = 1, \quad \psi \vee 1 = 1,$$

$$0 \supset \psi = 1, \quad \psi \supset 0 = \neg\psi, \quad 1 \supset \psi = \psi, \quad \psi \supset 1 = 1,$$

$$\neg 0 = 1, \quad \neg 1 = 0, \quad \neg\neg\psi = \psi.$$

Definitions

- Let φ be a propositional formula, $P = \{p_1, p_2, \ldots, p_n\}$ be the set of all variables of φ, and $P' = \{p_{i_1}, p_{i_2}, \ldots, p_{i_m}\}$ $(1 \leq m \leq n)$ be some subset of P.

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- Given $\sigma = \{\sigma_1, \sigma_2, \ldots, \sigma_m\} \subset E^m$, the conjunct $K^\sigma = \{{p_{i_1}}^{\sigma_1}, {p_{i_2}}^{\sigma_2}, \ldots, {p_{i_m}}^{\sigma_m}\}$ is called $\varphi - 1$ -determinative ($\varphi - 0$ -determinative) if assigning $\sigma_j (1 \leq j \leq m)$ to each p_{i_j} and successively using replacement-rules we obtain the value of φ (1 or 0) independently of the values of the remaining variables.

Definitions

- DNF $D = \{K_1, K_2, \ldots, K_j\}$ is called determinative DNF (DDNF) for φ if $\varphi = \mathrm{D}$ and every conjunct K_i $(1 \leq i \leq j)$ is 1-determinative for φ

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- The inference rule is *elimination rule* (ε-rule)

$$\frac{K_0 \cup \{p^0\}, \quad K_1 \cup \{p^1\}}{K_0 \cup \ K_1}$$

Definitions

- A finite sequence of conjuncts such that every conjunct in the sequence is one of the axioms of **EC** or is inferred from earlier conjuncts in the sequence by ε-rule is called a proof in **EC**. A DNF $D = \{K_1, K_2, \dots, K_l\}$ -tautological if by using ε-rule can be proved the empty conjunct $(\emptyset)$ from the axioms $\{K_1, K_2, \dots, K_l\}$.

Definitions

- Let φ be some propositional formula and p be some of its variable.Results of splitting method of formula φ by variable **p** ***(splinted variable)*** are the formlas φ[$\boldsymbol{p}^{\boldsymbol{\delta}}$] for every $\boldsymbol{\delta}$ from the set $\{0,1\}$, which are obtained from φ by assigning $\boldsymbol{\delta}$ to each occurrence of **p** and successively using replacement-rules.

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- The tree, which is constructed for formula φ by described method, we will call ***splitting tree of*** φ in future.

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- We can consider constant 1 as an axiom
- Inference rule $\dfrac{v[p^0], \quad v[p^1]}{v}$

Results

Theorem 1. The systems ***RC* , *RI*** and ***RJ*** are ***t-monotonous (l-monotonous)*** but neither of them is ***t-strong monotonous (l-strong monotonous).***

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Theorem 2. Each of the systems ***EC* , *EI, EJ*** and ***GS*** is neither ***t-monotonous (l-monotonous) and therefore*** not ***t-strong monotonous (l-strong monotonous).***

Thank you for attention