Cardinal invariants and ideal convergence

Jaroslav Šupina
joint work with R. Filipów and A. Kwela

Institute of Mathematics
Faculty of Science
P.J. Šafárik University in Košice

16$^{\text{th}}$ of August 2019

Ideals on natural numbers

A family $\mathcal{K} \subseteq \mathcal{P}(\omega)$ is called an ideal if

a) $B \in \mathcal{K}$ for any $B \subseteq A \in \mathcal{K}$,

b) $A \cup B \in \mathcal{K}$ for any $A, B \in \mathcal{K}$,

c) $\mathrm{Fin} = [\omega]^{<\omega} \subseteq \mathcal{K}$,

d) $\omega \notin \mathcal{K}$.

$\mathcal{I}$, $\mathcal{J}$, $\mathcal{K}$ are ideals in the following.

$\mathcal{K} \subseteq \mathcal{P}(\omega)$ $\qquad$ $\mathcal{K}^+ = \mathcal{P}(\omega) \setminus \mathcal{K}$

$\mathcal{A} \subseteq \mathcal{P}(\omega)$ $\qquad$ $\mathcal{A}^d = \{A \subseteq \omega : \omega \setminus A \in \mathcal{A}\}$

$\mathcal{F} \subseteq \mathcal{P}(\omega)$ is a filter if $\mathcal{F}^d$ is an ideal.

A maximal filter $\mathcal{U} \subseteq \mathcal{P}(\omega)$ is called an ultrafilter.

$\mathcal{D}_{\mathcal{K}} \subseteq {}^{\omega}\omega$ is a family of all $\mathcal{K}$-to-one functions.

Investigated bounding numbers, several authors

$$\mathfrak{b}(\mathcal{I},\mathcal{J},\mathcal{K}) = \min\{|A| :\ A \subseteq \mathcal{D}_{\mathcal{K}},\ (\forall \alpha \in \mathcal{D}_{\mathcal{J}})(\exists \beta \in A)\ \alpha \not\leq_{\mathcal{I}} \beta\}$$

$$= \mathfrak{b}(\mathcal{D}_{\mathcal{K}}, \mathcal{D}_{\mathcal{J}}, \geq_{\mathcal{I}})$$

$$\mathfrak{b}_{\mathrm{w}}(\mathcal{I},\mathcal{J},\mathcal{K}) = \min\{|A| :\ A \subseteq \mathcal{D}_{\mathcal{K}},\ (\forall \alpha \in \mathcal{D}_{\mathcal{J}})(\forall \varphi \in \mathcal{D}_{\mathcal{I}})(\exists \beta \in A)\ \alpha \not\leq_{\mathcal{I}} \varphi \circ \beta\}$$

$$= \mathfrak{b}(\mathcal{D}_{\mathcal{K}}, \mathcal{D}_{\mathcal{J}} \times \mathcal{D}_{\mathcal{I}}, \mathtt{R}_2 \circ \mathtt{L} \geq_{\mathcal{I}} \mathtt{R}_1)$$

$$\lambda(\mathcal{K},\mathcal{I}) = \min\{|A| :\ A \subseteq \mathcal{D}_{\mathcal{K}},\ (\forall \varphi \in {}^{\omega}\omega)(\exists \beta \in A)\ \mathtt{id} \not\leq_{\mathcal{I}} \varphi \circ \beta\}$$

$$= \mathfrak{b}(\mathcal{D}_{\mathcal{K}}, {}^{\omega}\omega, \mathtt{R} \circ \mathtt{L} \geq_{\mathcal{I}} \mathtt{id})$$

Investigated bounding numbers, several authors

$$\mathfrak{b}(\mathcal{I},\mathcal{J},\mathcal{K}) = \min\{|A| :\ A \subseteq \mathcal{D}_{\mathcal{K}},\ (\forall \alpha \in \mathcal{D}_{\mathcal{J}})(\exists \beta \in A)\ \alpha \not\leq_{\mathcal{I}} \beta\}$$

$$= \mathfrak{b}(\mathcal{D}_{\mathcal{K}}, \mathcal{D}_{\mathcal{J}}, \geq_{\mathcal{I}})$$

$$\mathfrak{b}_{\mathrm{w}}(\mathcal{I},\mathcal{J},\mathcal{K}) = \min\{|A| :\ A \subseteq \mathcal{D}_{\mathcal{K}},\ (\forall \alpha \in \mathcal{D}_{\mathcal{J}})(\forall \varphi \in \mathcal{D}_{\mathcal{I}})(\exists \beta \in A)\ \alpha \not\leq_{\mathcal{I}} \varphi \circ \beta\}$$

$$= \mathfrak{b}(\mathcal{D}_{\mathcal{K}}, \mathcal{D}_{\mathcal{J}} \times \mathcal{D}_{\mathcal{I}}, \mathtt{R}_2 \circ \mathtt{L} \geq_{\mathcal{I}} \mathtt{R}_1)$$

$$\lambda(\mathcal{K},\mathcal{I}) = \min\{|A| :\ A \subseteq \mathcal{D}_{\mathcal{K}},\ (\forall \varphi \in {}^{\omega}\omega)(\exists \beta \in A)\ \mathtt{id} \not\leq_{\mathcal{I}} \varphi \circ \beta\}$$

$$= \mathfrak{b}(\mathcal{D}_{\mathcal{K}}, {}^{\omega}\omega, \mathtt{R} \circ \mathtt{L} \geq_{\mathcal{I}} \mathtt{id})$$

Investigated bounding numbers, several authors

$$\begin{aligned}\mathfrak{b}(\mathcal{I},\mathcal{J},\mathcal{K}) &= \min\{|A| :\ A \subseteq \mathcal{D}_{\mathcal{K}},\ (\forall \alpha \in \mathcal{D}_{\mathcal{J}})(\exists \beta \in A)\ \alpha \not\leq_{\mathcal{I}} \beta\} \\ &= \mathfrak{b}(\mathcal{D}_{\mathcal{K}}, \mathcal{D}_{\mathcal{J}}, \geq_{\mathcal{I}})\end{aligned}$$

$$\begin{aligned}\mathfrak{b}_{\mathrm{w}}(\mathcal{I},\mathcal{J},\mathcal{K}) &= \min\{|A| :\ A \subseteq \mathcal{D}_{\mathcal{K}},\ (\forall \alpha \in \mathcal{D}_{\mathcal{J}})(\forall \varphi \in \mathcal{D}_{\mathcal{I}})(\exists \beta \in A)\ \alpha \not\leq_{\mathcal{I}} \varphi \circ \beta\} \\ &= \mathfrak{b}(\mathcal{D}_{\mathcal{K}}, \mathcal{D}_{\mathcal{J}} \times \mathcal{D}_{\mathcal{I}}, \mathtt{R}_2 \circ \mathtt{L} \geq_{\mathcal{I}} \mathtt{R}_1)\end{aligned}$$

$$\begin{aligned}\lambda(\mathcal{K},\mathcal{I}) &= \min\{|A| :\ A \subseteq \mathcal{D}_{\mathcal{K}},\ (\forall \varphi \in {}^{\omega}\omega)(\exists \beta \in A)\ \mathtt{id} \not\leq_{\mathcal{I}} \varphi \circ \beta\} \\ &= \mathfrak{b}(\mathcal{D}_{\mathcal{K}}, {}^{\omega}\omega, \mathtt{R} \circ \mathtt{L} \geq_{\mathcal{I}} \mathtt{id})\end{aligned}$$

Investigated bounding numbers, several authors

$$\mathfrak{b}(\mathcal{I},\mathcal{J},\mathcal{K}) = \min\{|A| :\ A \subseteq \mathcal{D}_{\mathcal{K}},\ (\forall \alpha \in \mathcal{D}_{\mathcal{J}})(\exists \beta \in A)\ \alpha \not\leq_{\mathcal{I}} \beta\}$$
$$= \mathfrak{b}(\mathcal{D}_{\mathcal{K}}, \mathcal{D}_{\mathcal{J}}, \geq_{\mathcal{I}})$$

$$\mathfrak{b}_{\mathrm{w}}(\mathcal{I},\mathcal{J},\mathcal{K}) = \min\{|A| :\ A \subseteq \mathcal{D}_{\mathcal{K}},\ (\forall \alpha \in \mathcal{D}_{\mathcal{J}})(\forall \varphi \in \mathcal{D}_{\mathcal{I}})(\exists \beta \in A)\ \alpha \not\leq_{\mathcal{I}} \varphi \circ \beta\}$$
$$= \mathfrak{b}(\mathcal{D}_{\mathcal{K}}, \mathcal{D}_{\mathcal{J}} \times \mathcal{D}_{\mathcal{I}}, \mathtt{R}_2 \circ \mathtt{L} \geq_{\mathcal{I}} \mathtt{R}_1)$$

$$\lambda(\mathcal{K},\mathcal{I}) = \min\{|A| :\ A \subseteq \mathcal{D}_{\mathcal{K}},\ (\forall \varphi \in {}^{\omega}\omega)(\exists \beta \in A)\ \mathtt{id} \not\leq_{\mathcal{I}} \varphi \circ \beta\}$$
$$= \mathfrak{b}(\mathcal{D}_{\mathcal{K}}, {}^{\omega}\omega, \mathtt{R} \circ \mathtt{L} \geq_{\mathcal{I}} \mathtt{id})$$

Investigated bounding numbers, several authors

$$\mathfrak{b}(\mathcal{I},\mathcal{J},\mathcal{K}) = \min\{|A| :\ A \subseteq \mathcal{D}_{\mathcal{K}},\ (\forall \alpha \in \mathcal{D}_{\mathcal{J}})(\exists \beta \in A)\ \alpha \not\leq_{\mathcal{I}} \beta\}$$
$$= \mathfrak{b}(\mathcal{D}_{\mathcal{K}}, \mathcal{D}_{\mathcal{J}}, \geq_{\mathcal{I}})$$

$$\mathfrak{b}_{\mathrm{w}}(\mathcal{I},\mathcal{J},\mathcal{K}) = \min\{|A| :\ A \subseteq \mathcal{D}_{\mathcal{K}},\ (\forall \alpha \in \mathcal{D}_{\mathcal{J}})(\forall \varphi \in \mathcal{D}_{\mathcal{I}})(\exists \beta \in A)\ \alpha \not\leq_{\mathcal{I}} \varphi \circ \beta\}$$
$$= \mathfrak{b}(\mathcal{D}_{\mathcal{K}}, \mathcal{D}_{\mathcal{J}} \times \mathcal{D}_{\mathcal{I}}, \mathtt{R}_2 \circ \mathtt{L} \geq_{\mathcal{I}} \mathtt{R}_1)$$

$$\lambda(\mathcal{K},\mathcal{I}) = \min\{|A| :\ A \subseteq \mathcal{D}_{\mathcal{K}},\ (\forall \varphi \in {}^{\omega}\omega)(\exists \beta \in A)\ \mathtt{id} \not\leq_{\mathcal{I}} \varphi \circ \beta\}$$
$$= \mathfrak{b}(\mathcal{D}_{\mathcal{K}}, {}^{\omega}\omega, \mathtt{R} \circ \mathtt{L} \geq_{\mathcal{I}} \mathtt{id})$$

Investigated bounding numbers, several authors

$$\begin{aligned}\mathfrak{b}(\mathcal{I},\mathcal{J},\mathcal{K}) &= \min\{|A| : A \subseteq \mathcal{D}_{\mathcal{K}},\ (\forall\alpha \in \mathcal{D}_{\mathcal{J}})(\exists\beta \in A)\ \alpha \not\leq_{\mathcal{I}} \beta\} \\ &= \mathfrak{b}(\mathcal{D}_{\mathcal{K}}, \mathcal{D}_{\mathcal{J}}, \geq_{\mathcal{I}})\end{aligned}$$

$$\begin{aligned}\mathfrak{b}_{\mathrm{w}}(\mathcal{I},\mathcal{J},\mathcal{K}) &= \min\{|A| : A \subseteq \mathcal{D}_{\mathcal{K}},\ (\forall\alpha \in \mathcal{D}_{\mathcal{J}})(\forall\varphi \in \mathcal{D}_{\mathcal{I}})(\exists\beta \in A)\ \alpha \not\leq_{\mathcal{I}} \varphi \circ \beta\} \\ &= \mathfrak{b}(\mathcal{D}_{\mathcal{K}}, \mathcal{D}_{\mathcal{J}} \times \mathcal{D}_{\mathcal{I}}, \mathtt{R}_2 \circ \mathtt{L} \geq_{\mathcal{I}} \mathtt{R}_1)\end{aligned}$$

$$\begin{aligned}\lambda(\mathcal{K},\mathcal{I}) &= \min\{|A| : A \subseteq \mathcal{D}_{\mathcal{K}},\ (\forall\varphi \in {}^{\omega}\omega)(\exists\beta \in A)\ \mathtt{id} \not\leq_{\mathcal{I}} \varphi \circ \beta\} \\ &= \mathfrak{b}(\mathcal{D}_{\mathcal{K}}, {}^{\omega}\omega, \mathtt{R} \circ \mathtt{L} \geq_{\mathcal{I}} \mathtt{id})\end{aligned}$$

Investigated bounding numbers, several authors

$$\begin{aligned}\mathfrak{b}(\mathcal{I},\mathcal{J},\mathcal{K}) &= \min\{|A| :\ A \subseteq \mathcal{D}_{\mathcal{K}},\ (\forall \alpha \in \mathcal{D}_{\mathcal{J}})(\exists \beta \in A)\ \alpha \not\leq_{\mathcal{I}} \beta\} \\ &= \mathfrak{b}(\mathcal{D}_{\mathcal{K}}, \mathcal{D}_{\mathcal{J}}, \geq_{\mathcal{I}})\end{aligned}$$

$$\begin{aligned}\mathfrak{b}_{\mathrm{w}}(\mathcal{I},\mathcal{J},\mathcal{K}) &= \min\{|A| :\ A \subseteq \mathcal{D}_{\mathcal{K}},\ (\forall \alpha \in \mathcal{D}_{\mathcal{J}})(\forall \varphi \in \mathcal{D}_{\mathcal{I}})(\exists \beta \in A)\ \alpha \not\leq_{\mathcal{I}} \varphi \circ \beta\} \\ &= \mathfrak{b}(\mathcal{D}_{\mathcal{K}}, \mathcal{D}_{\mathcal{J}} \times \mathcal{D}_{\mathcal{I}}, \mathtt{R}_2 \circ \mathtt{L} \geq_{\mathcal{I}} \mathtt{R}_1)\end{aligned}$$

$$\begin{aligned}\lambda(\mathcal{K},\mathcal{I}) &= \min\{|A| :\ A \subseteq \mathcal{D}_{\mathcal{K}},\ (\forall \varphi \in {}^{\omega}\omega)(\exists \beta \in A)\ \mathtt{id} \not\leq_{\mathcal{I}} \varphi \circ \beta\} \\ &= \mathfrak{b}(\mathcal{D}_{\mathcal{K}}, {}^{\omega}\omega, \mathtt{R} \circ \mathtt{L} \geq_{\mathcal{I}} \mathtt{id})\end{aligned}$$

wQN-space and QN-space

$C_p(X)$ of a topological space X is a $[\Gamma_{\mathbf{0}}, Q_{\mathbf{0}}]$-space if

wQN-space and QN-space

$\mathrm{C}_p(X)$ of a topological space X is a $[\Gamma_{\mathbf{0}}, \mathrm{Q}_{\mathbf{0}}]$-space if

(a) for every $f_n \in \mathrm{C}_p(X)$ such that $f_n \to \mathbf{0}$,

(b) there is $\varphi \in {}^{\omega}\omega$,

(c) there is $\varepsilon_k \in [0, +\infty)$ such that $\varepsilon_k \to 0$,

(d) $|f_{\varphi(k)}(x)| < \varepsilon_k$ for all but finitely many k in every $x \in X$.

wQN-space and QN-space

$\mathrm{C}_p(X)$ of a topological space X is a $[\Gamma_{\mathbf{0}}, \mathrm{Q}_{\mathbf{0}}]$-space if

(a) for every $f_n \in \mathrm{C}_p(X)$ such that $f_n \to \mathbf{0}$,

(b) there is $\varphi \in {}^\omega\omega$,

(c) there is $\varepsilon_k \in [0, +\infty)$ such that $\varepsilon_k \to 0$,

(d) $|f_{\varphi(k)}(x)| < \varepsilon_k$ for all but finitely many k in every $x \in X$.

(a) pointwise convergence, (c)+(d) quasi-normal convergence

wQN-space and QN-space

$\mathrm{C}_p(X)$ of a topological space X is a $[\Gamma_{\mathbf{0}}, \mathrm{Q}_{\mathbf{0}}]$-space if

(a) for every $f_n \in \mathrm{C}_p(X)$ such that $f_n \to \mathbf{0}$,

(b) there is $\varphi \in {}^\omega\omega$,

(c) there is $\varepsilon_k \in [0, +\infty)$ such that $\varepsilon_k \to 0$,

(d) $|f_{\varphi(k)}(x)| < \varepsilon_k$ for all but finitely many k in every $x \in X$.

(a) pointwise convergence, (c)+(d) quasi-normal convergence

$\mathrm{C}_p(X)$ of a topological space X is a $[\Gamma_{\mathbf{0}}, \mathrm{Q}_{\mathbf{0}}]^{\mathrm{id}}$-space if

(a) for every $f_n \in \mathrm{C}_p(X)$ such that $f_n \to \mathbf{0}$,

(c) there is $\varepsilon_n \in [0, +\infty)$ such that $\varepsilon_n \to 0$,

(d) $|f_n(x)| < \varepsilon_n$ for all but finitely many n in every $x \in X$.

More about wQN-space and QN-space, some samples

- Bukovský L., Recław I. and Repický M., *Spaces not distinguishing pointwise and quasinormal convergence of real functions,* Topology Appl. **41** (1991), 25–40.
- Recław I., *Metric spaces not distinguishing pointwise and quasinormal convergence of real functions,* Bull. Acad. Polon. Sci. **45** (1997), 287–289.
- Scheepers M., $C_p(X)$ *and Archangel'skiĭ's* α_i*-spaces,* Topology Appl. **45** (1998), 265–275.
- Bukovský L., Recław I. and Repický M., *Spaces not distinguishing convergences of real–valued functions,* Topology Appl. **112** (2001), 13–40.
- Fremlin D.H., SSP *and* wQN, Notes of 14.01.2003, https://www1.essex.ac.uk/maths/people/fremlin/preprints.htm.
- Haleš J., *On Scheepers' conjecture,* Acta Univ. Carolinae Math. Phys. **46** (2005), 27–31.
- Bukovský L. and Haleš J., QN*-spaces,* wQN*-spaces and covering properties,* Topology Appl. **154** (2007), 848–858.
- Sakai M., *The sequence selection properties of* $C_p(X)$, Topology Appl. **154** (2007), 552–560.
- Ohta H. and Sakai M., *Sequences of semicontinuous functions accompanying continuous functions*, Topology Appl. **156** (2009), 2683–2691.
- Bukovský L., The Structure of the Real Line, Monogr. Mat., Springer-Birkhauser, Basel, 2011.
- Tsaban B. and Zdomskyy L., *Hereditary Hurewicz spaces and Arhangel'skiĭ sheaf amalgamations,* J. Eur. Math. Soc. (JEMS), **14** (2012), 353–372.
- Šupina J., *On Ohta–Sakai's properties of a topological space*, Topology Appl. **190** (2015), 119–134.

⋮

Critical cardinality

$\mathrm{non}(\texttt{Property})$ is the least κ such that

- if X is a perfectly normal topological space of size less than κ then $\mathrm{C}_p(X)$ possesses $\texttt{Property}$,
- there is a perfectly normal topological space X of size κ such that $\mathrm{C}_p(X)$ does not possess $\texttt{Property}$.

$$\mathrm{non}(\left[\begin{smallmatrix}\Gamma_{\mathbf{O}}\\ \mathrm{Q}_{\mathbf{O}}\end{smallmatrix}\right]) = \mathrm{non}(\left[\begin{smallmatrix}\Gamma_{\mathbf{O}}\\ \mathrm{Q}_{\mathbf{O}}\end{smallmatrix}\right]^{\mathrm{id}}) = \mathfrak{b}$$

wQN-space and QN-space

$C_p(X)$ is a $[\Gamma_{\mathbf{0}}, Q_{\mathbf{0}}]$-space if

(a) for every $f_n \in C_p(X)$ such that $f_n \to \mathbf{0}$,

(b) there is $\varphi \in {}^{\omega}\omega$,

(c) there is $\varepsilon_k \in [0, +\infty)$ such that $\varepsilon_k \to 0$,

(d) $|f_{\varphi(k)}(x)| < \varepsilon_k$ for all but finitely many k in every $x \in X$.

$C_p(X)$ is a $[\Gamma_{\mathbf{0}}, Q_{\mathbf{0}}]^{\mathrm{id}}$-space if

(a) for every $f_n \in C_p(X)$ such that $f_n \to \mathbf{0}$,

(c) there is $\varepsilon_n \in [0, +\infty)$ such that $\varepsilon_n \to 0$,

(d) $|f_n(x)| < \varepsilon_n$ for all but finitely many n in every $x \in X$.

wQN-space and QN-space

$\mathrm{C}_p(X)$ is a $[\Gamma_{\mathbf{0}}, \mathrm{Q}_{\mathbf{0}}]$-space if

(a) for every $f_n \in \mathrm{C}_p(X)$ such that $\{n : |f_n(x)| \geq \varepsilon\} \in \mathrm{Fin}$ for every $\varepsilon > 0$,

(b) there is $\varphi \in {}^{\omega}\omega$,

(c) there is $\varepsilon_k \in [0, +\infty)$ such that $\{k : \varepsilon_k \geq \varepsilon\} \in \mathrm{Fin}$ for every $\varepsilon > 0$,

(d) $\{k : |f_{\varphi(k)}(x)| \geq \varepsilon_k\} \in \mathrm{Fin}$ in every $x \in X$.

$\mathrm{C}_p(X)$ is a $[\Gamma_{\mathbf{0}}, \mathrm{Q}_{\mathbf{0}}]^{\mathrm{id}}$-space if

(a) for every $f_n \in \mathrm{C}_p(X)$ such that $\{n : |f_n(x)| \geq \varepsilon\} \in \mathrm{Fin}$ for every $\varepsilon > 0$,

(c) there is $\varepsilon_n \in [0, +\infty)$ such that $\{n : \varepsilon_n \geq \varepsilon\} \in \mathrm{Fin}$ for every $\varepsilon > 0$,

(d) $\{n : |f_n(x)| \geq \varepsilon_n\} \in \mathrm{Fin}$ in every $x \in X$.

A difficult road to ideal wQN-space and QN-space

- R. Filipów and P. Szuca, 2012

(c) $\{n: \ \varepsilon_n \geq \varepsilon\} \in \mathrm{Fin}$	(d) $\{k: \ |f_n(x)| \geq \varepsilon_n\} \in \mathcal{I}$

Baire Hierarchy

A difficult road to ideal wQN-space and QN-space

- R. Filipów and P. Szuca, 2012

(c) $\{n : \varepsilon_n \geq \varepsilon\} \in \mathrm{Fin}$	(d) $\{k : \|f_n(x)\| \geq \varepsilon_n\} \in \mathcal{I}$

Baire Hierarchy

- P. Das and D. Chandra, 2013

$\{n : \|f_n(x)\| \geq \varepsilon\} \in \mathrm{Fin}$	$\{n : \varepsilon_n \geq \varepsilon\} \in \mathcal{I}$	$\{k : \|f_{\varphi(k)}(x)\| \geq \varepsilon_k\} \in \mathcal{I}$
$\{n : \|f_n(x)\| \geq \varepsilon\} \in \mathrm{Fin}$	$\{n : \varepsilon_n \geq \varepsilon\} \in \mathcal{I}$	$\{n : \|f_n(x)\| \geq \varepsilon_n\} \in \mathcal{I}$

A difficult road to ideal wQN-space and QN-space

- R. Filipów and P. Szuca, 2012

(c) $\{n : \varepsilon_n \geq \varepsilon\} \in \mathrm{Fin}$	(d) $\{k : \|f_n(x)\| \geq \varepsilon_n\} \in \mathcal{I}$

Baire Hierarchy

- P. Das and D. Chandra, 2013

$\{n : \|f_n(x)\| \geq \varepsilon\} \in \mathrm{Fin}$	$\{n : \varepsilon_n \geq \varepsilon\} \in \mathcal{I}$	$\{k : \|f_{\varphi(k)}(x)\| \geq \varepsilon_k\} \in \mathcal{I}$
$\{n : \|f_n(x)\| \geq \varepsilon\} \in \mathrm{Fin}$	$\{n : \varepsilon_n \geq \varepsilon\} \in \mathcal{I}$	$\{n : \|f_n(x)\| \geq \varepsilon_n\} \in \mathcal{I}$

- R. Filipów and M. Staniszewski, 2014

$\{n : \varepsilon_n \geq \varepsilon\} \in \mathcal{I}$	$\{n : \|f_n(x)\| \geq \varepsilon_n\} \in \mathcal{J}$

Just $\mathcal{J} \subseteq \mathcal{I}$ implies $\mathcal{I}$-pointwise convergence.

A difficult road to ideal wQN-space and QN-space

- R. Filipów and M. Staniszewski, 2015

$\{n : \|f_n(x)\| \geq \varepsilon\} \in \mathcal{I}$	$\{n : \varepsilon_n \geq \varepsilon\} \in \mathcal{J}$	$\{n : \|f_n(x)\| \geq \varepsilon_n\} \in \mathcal{I}$

A difficult road to ideal wQN-space and QN-space

- R. Filipów and M. Staniszewski, 2015

$\{n : \ |f_n(x)| \geq \varepsilon\} \in \mathcal{I}$	$\{n : \ \varepsilon_n \geq \varepsilon\} \in \mathcal{J}$	$\{n : \ |f_n(x)| \geq \varepsilon_n\} \in \mathcal{I}$

Theorem

If $\mathrm{W}(\mathcal{I},\mathcal{J})$ *does not hold, then any topological space is an* $[\mathcal{I}\text{-}\Gamma_{\mathbf{0}}, \mathcal{I}\mathcal{J}\text{-}\mathrm{Q}_{\mathbf{0}}]^{\mathrm{id}}$*-space.*

A difficult road to ideal wQN-space and QN-space

- R. Filipów and M. Staniszewski, 2015

$\{n : \|f_n(x)\| \geq \varepsilon\} \in \mathcal{I}$	$\{n : \varepsilon_n \geq \varepsilon\} \in \mathcal{J}$	$\{n : \|f_n(x)\| \geq \varepsilon_n\} \in \mathcal{I}$

Theorem

If $\mathrm{W}(\mathcal{I},\mathcal{J})$ *does not hold, then any topological space is an* $[\mathcal{I}\text{-}\Gamma_{\mathbf{0}}, \mathcal{I}\mathcal{J}\text{-}\mathrm{Q}_{\mathbf{0}}]^{\mathrm{id}}$*-space.*

$$\mathrm{non}\left(\begin{bmatrix}\mathcal{I}\text{-}\Gamma_{\mathbf{0}} \\ \mathcal{I}\mathcal{J}\text{-}\mathrm{Q}_{\mathbf{0}}\end{bmatrix}^{\mathrm{id}}\right) = \min(\{\mathfrak{c}^+\} \cup$$
$$\left\{|\mathcal{S}| : \mathcal{S} \subseteq {}^{\omega}\mathcal{I}^{\mathrm{m.d.}} \wedge (\forall \{A_n : n \in \omega\} \in \mathcal{P}_{\mathcal{J}})(\exists s \in \mathcal{S}) \bigcup_{n \in \omega} (A_n \cap \bigcup_{i \leq n} s(i)) \in \mathcal{I}^+\right\}).$$

A difficult road to ideal wQN-space and QN-space

- J.Š., 2016

$\{n : \vert f_n(x)\vert \geq \varepsilon\} \in \mathcal{K}$	$\{n : \varepsilon_n \geq \varepsilon\} \in \mathcal{I}$	$\{n : \vert f_n(x)\vert \geq \varepsilon_n\} \in \mathcal{I}$

A difficult road to ideal wQN-space and QN-space

- J.Š., 2016

$\{n : \lvert f_n(x)\rvert \geq \varepsilon\} \in \mathcal{K}$	$\{n : \varepsilon_n \geq \varepsilon\} \in \mathcal{I}$	$\{n : \lvert f_n(x)\rvert \geq \varepsilon_n\} \in \mathcal{I}$

Theorem

If $\mathcal{I}$ is not a weak $\mathrm{P}(\mathcal{K})$-ideal then any topological space is a $[\mathcal{K}\text{-}\Gamma_{\mathbf{0}}, \mathcal{I}\mathcal{I}\text{-}\mathrm{Q}_{\mathbf{0}}]^{\mathrm{id}}$-space.

A difficult road to ideal wQN-space and QN-space

- J.Š., 2016

$\{n : \ |f_n(x)| \geq \varepsilon\} \in \mathcal{K}$	$\{n : \ \varepsilon_n \geq \varepsilon\} \in \mathcal{I}$	$\{n : \ |f_n(x)| \geq \varepsilon_n\} \in \mathcal{I}$

Theorem

If $\mathcal{I}$ is not a weak $\mathrm{P}(\mathcal{K})$-ideal then any topological space is a $[\mathcal{K}\text{-}\Gamma_{\mathbf{0}}, \mathcal{I}\mathcal{I}\text{-}\mathrm{Q}_{\mathbf{0}}]^{\mathrm{id}}$-space.

Corollary

Let $\mathcal{U}$ be an ultrafilter which is not a P-point. Then any topological space is a $[\Gamma_{\mathbf{0}}, \mathcal{U}\mathcal{U}\text{-}\mathrm{Q}_{\mathbf{0}}]^{\mathrm{id}}$-space.

A difficult road to ideal wQN-space and QN-space

- J.Š., 2016

$\{n : \ |f_n(x)| \geq \varepsilon\} \in \mathcal{K}$	$\{n : \ \varepsilon_n \geq \varepsilon\} \in \mathcal{I}$	$\{n : \ |f_n(x)| \geq \varepsilon_n\} \in \mathcal{I}$

Theorem

If $\mathcal{I}$ is not a weak $\mathrm{P}(\mathcal{K})$-ideal then any topological space is a $[\mathcal{K}\text{-}\Gamma_{\mathbf{0}}, \mathcal{I}\mathcal{I}\text{-}\mathrm{Q}_{\mathbf{0}}]^{\mathrm{id}}$-space.

Corollary

Let $\mathcal{U}$ be an ultrafilter which is not a P-point. Then any topological space is a $[\Gamma_{\mathbf{0}}, \mathcal{U}\mathcal{U}\text{-}\mathrm{Q}_{\mathbf{0}}]^{\mathrm{id}}$-space.

$$\mathrm{non}\left(\left[{\mathcal{K}\text{-}\Gamma_{\mathbf{0}} \atop \mathcal{I}\mathcal{I}\text{-}\mathrm{Q}_{\mathbf{0}}}\right]^{\mathrm{id}}\right) = \min(\{\mathfrak{c}^+\} \cup$$
$$\left\{|\mathcal{S}| : \ \mathcal{S} \subseteq {}^{\omega}\mathcal{K} \wedge (\forall \{A_n : \ n \in \omega\} \in \mathcal{P}_{\mathcal{I}})(\exists s \in \mathcal{S}) \bigcup_{n \in \omega} (A_n \cap s(n)) \in \mathcal{I}^+\right\}).$$

A difficult road to ideal wQN-space and QN-space

- L. Bukovský, P. Das and J.Š., 2017

$$\boxed{\{n:\ |f_n(x)| \geq \varepsilon\} \in \mathcal{K}} \quad \boxed{} \quad \boxed{\left\{k:\ |f_{\varphi(k)}(x)| \geq 2^{-k}\right\} \in \mathcal{I}}$$

selection principles

A difficult road to ideal wQN-space and QN-space

- L. Bukovský, P. Das and J.Š., 2017

$$\boxed{\{n :\ |f_n(x)| \geq \varepsilon\} \in \mathcal{K}} \quad \boxed{} \quad \boxed{\left\{k :\ |f_{\varphi(k)}(x)| \geq 2^{-k}\right\} \in \mathcal{I}}$$

selection principles

- M. Staniszewski, 2017

$$\boxed{\{n :\ |f_n(x)| \geq \varepsilon\} \in \mathcal{K}} \quad \boxed{\{n :\ \varepsilon_n \geq \varepsilon\} \in \mathcal{J}} \quad \boxed{\{n :\ |f_n(x)| \geq \varepsilon_n\} \in \mathcal{I}}$$

$$\mathrm{non}\left(\left[\begin{smallmatrix}\mathcal{K}\text{-}\Gamma_{\mathbf{0}} \\ \mathcal{I}\mathcal{J}\text{-}\mathrm{Q}_{\mathbf{0}}\end{smallmatrix}\right]^{\mathrm{id}}\right) = \min(\{\mathfrak{c}^+\} \cup$$
$$\left\{|\mathcal{S}| :\ \mathcal{S} \subseteq {}^{\omega}\mathcal{K}^{\mathrm{m.d.}} \wedge (\forall \{A_n :\ n \in \omega\} \in \mathcal{P}_{\mathcal{J}})(\exists s \in \mathcal{S}) \bigcup_{n\in\omega} (A_n \cap \bigcup_{i \leq n} s(i)) \in \mathcal{I}^+\right\}).$$

A difficult road to ideal wQN-space and QN-space

- L. Bukovský, P. Das and J.Š., 2017

$\{n:\ \lvert f_n(x)\rvert \geq \varepsilon\} \in \mathcal{K}$		$\left\{k:\ \lvert f_{\varphi(k)}(x)\rvert \geq 2^{-k}\right\} \in \mathcal{I}$

selection principles

- M. Staniszewski, 2017

$\{n:\ \lvert f_n(x)\rvert \geq \varepsilon\} \in \mathcal{K}$	$\{n:\ \varepsilon_n \geq \varepsilon\} \in \mathcal{J}$	$\{n:\ \lvert f_n(x)\rvert \geq \varepsilon_n\} \in \mathcal{I}$

$$\mathrm{non}\left(\left[{}^{\mathcal{K}\text{-}\Gamma_{\mathbf{0}}}_{\mathcal{I}\mathcal{J}\text{-}\mathrm{Q}_{\mathbf{0}}}\right]^{\mathrm{id}}\right) = \min(\{\mathfrak{c}^+\} \cup \left\{\lvert\mathcal{S}\rvert:\ \mathcal{S} \subseteq {}^{\omega}\mathcal{K}^{\mathrm{m.d.}} \wedge (\forall\,\{A_n:\ n\in\omega\} \in \mathcal{P}_{\mathcal{J}})(\exists s\in\mathcal{S}) \bigcup_{n\in\omega}(A_n \cap \bigcup_{i\leq n} s(i)) \in \mathcal{I}^+\right\}).$$

- A. Kwela, 2018

$\{n:\ \lvert f_n(x)\rvert \geq \varepsilon\} \in \mathrm{Fin}$	$\{n:\ \varepsilon_n \geq \varepsilon\} \in \mathcal{I}$	$\left\{k:\ \lvert f_{\varphi(k)}(x)\rvert \geq \varepsilon_k\right\} \in \mathcal{I}$

$$\mathrm{non}\left(\left[{}^{\Gamma_{\mathbf{0}}}_{\mathcal{I}\mathcal{I}\text{-}\mathrm{Q}_{\mathbf{0}}}\right]\right) = \min(\{\mathfrak{c}^+\} \cup \{\lvert\mathcal{S}\rvert : \mathcal{S} \subseteq {}^{\omega}\mathrm{Fin} \wedge (\forall B \in [\omega]^{\omega})(\forall\,\{D_n:\ n\in\omega\} \in \mathcal{P}_{\mathcal{J}})(\exists s\in\mathcal{S}) \bigcup_{n\in\omega}(D_n \cap \mathrm{e}_B^{-1}[s(n)]) \in \mathcal{I}^+\}).$$

A difficult road to ideal wQN-space and QN-space

- V. Šottová and J.Š., 2019

$$\boxed{\{n : \ |f_n(x)| \geq \varepsilon\} \in \mathcal{K}} \quad \boxed{} \quad \boxed{\left\{k : \ |f_{\varphi(k)}(x)| \geq 2^{-k}\right\} \in \mathcal{I}}$$

$$\mathrm{non}(\begin{bmatrix}\mathcal{K}\text{-}\Gamma_{\mathbf{0}} \\ \mathcal{I}\text{-}\mathrm{Q}^{\mathrm{s}}_{\mathbf{0}}\end{bmatrix}) = \min\left\{|\mathcal{S}| : \ \mathcal{S} \subseteq {}^{\omega}\mathcal{K}, \ (\forall\varphi \in \mathcal{D}_{\mathcal{I}})(\exists s \in \mathcal{S}) \ \{n : \varphi(n) \in s(n)\} \in \mathcal{I}^{+}\right\}$$

A difficult road to ideal wQN-space and QN-space

- V. Šottová and J.Š., 2019

$$\boxed{\{n:\ |f_n(x)|\geq\varepsilon\}\in\mathcal{K}}\ \boxed{}\ \boxed{\left\{k:\ |f_{\varphi(k)}(x)|\geq 2^{-k}\right\}\in\mathcal{I}}$$

$$\mathrm{non}(\begin{bmatrix}\mathcal{K}\text{-}\Gamma_{\mathbf{0}}\\ \mathcal{I}\text{-}\mathrm{Q}^{\mathrm{s}}_{\mathbf{0}}\end{bmatrix}) = \min\left\{|\mathcal{S}|:\ \mathcal{S}\subseteq{}^{\omega}\mathcal{K},\ (\forall\varphi\in\mathcal{D}_{\mathcal{I}})(\exists s\in\mathcal{S})\ \{n:\varphi(n)\in s(n)\}\in\mathcal{I}^{+}\right\}$$

- R. Filipów and A. Kwela, M. Repický, 2019

$$\mathrm{non}(\begin{bmatrix}\mathcal{K}\text{-}\Gamma_{\mathbf{0}}\\ \mathcal{I}\mathcal{J}\text{-}\mathrm{Q}_{\mathbf{0}}\end{bmatrix}^{\mathrm{id}}) = \min\{|A|:\ A\subseteq\mathcal{D}_{\mathcal{K}},\ (\forall\alpha\in\mathcal{D}_{\mathcal{J}})(\exists\beta\in A)\ \alpha\not\leq_{\mathcal{I}}\beta\}$$

A difficult road to ideal wQN-space and QN-space

- V. Šottová and J.Š., 2019

$$\boxed{\{n : |f_n(x)| \geq \varepsilon\} \in \mathcal{K}} \quad \boxed{} \quad \boxed{\{k : |f_{\varphi(k)}(x)| \geq 2^{-k}\} \in \mathcal{I}}$$

$$\mathrm{non}(\begin{bmatrix} \mathcal{K}\text{-}\Gamma_{\mathbf{0}} \\ \mathcal{I}\text{-}\mathrm{Q}^{\mathrm{s}}_{\mathbf{0}} \end{bmatrix}) = \min\left\{|\mathcal{S}| : \mathcal{S} \subseteq {}^{\omega}\mathcal{K}, (\forall \varphi \in \mathcal{D}_{\mathcal{I}})(\exists s \in \mathcal{S}) \ \{n : \varphi(n) \in s(n)\} \in \mathcal{I}^+\right\}$$

- R. Filipów and A. Kwela, M. Repický, 2019

$$\mathrm{non}(\begin{bmatrix} \mathcal{K}\text{-}\Gamma_{\mathbf{0}} \\ \mathcal{I}\mathcal{J}\text{-}\mathrm{Q}_{\mathbf{0}} \end{bmatrix}^{\mathrm{id}}) = \min\{|A| : A \subseteq \mathcal{D}_{\mathcal{K}}, (\forall \alpha \in \mathcal{D}_{\mathcal{J}})(\exists \beta \in A) \ \alpha \not\leq_{\mathcal{I}} \beta\}$$

- M. Repický, 2019

$$\mathrm{non}(\begin{bmatrix} \mathcal{K}\text{-}\Gamma_{\mathbf{0}} \\ \mathcal{I}\mathcal{J}\text{-}\mathrm{Q}_{\mathbf{0}} \end{bmatrix}) = \min\{|A| : A \subseteq \mathcal{D}_{\mathcal{K}}, (\forall \alpha \in \mathcal{D}_{\mathcal{J}})(\forall \varphi \in \mathcal{D}_{\mathcal{I}})(\exists \beta \in A) \ \alpha \not\leq_{\mathcal{I}} \varphi \circ \beta\}$$

A difficult road to ideal wQN-space and QN-space

- V. Šottová and J.Š., 2019

$$\boxed{\{n:\ |f_n(x)|\geq\varepsilon\}\in\mathcal{K}}\ \boxed{}\ \boxed{\{k:\ |f_{\varphi(k)}(x)|\geq 2^{-k}\}\in\mathcal{I}}$$

$$\mathrm{non}(\begin{bmatrix}\mathcal{K}\text{-}\Gamma_{\mathbf{0}}\\ \mathcal{I}\text{-}\mathrm{Q}^{\mathrm{s}}_{\mathbf{0}}\end{bmatrix})=\min\{|\mathcal{S}|:\ \mathcal{S}\subseteq{}^{\omega}\mathcal{K},\ (\forall\varphi\in\mathcal{D}_{\mathcal{I}})(\exists s\in\mathcal{S})\ \{n:\varphi(n)\in s(n)\}\in\mathcal{I}^{+}\}$$

- R. Filipów and A. Kwela, M. Repický, 2019

$$\mathrm{non}(\begin{bmatrix}\mathcal{K}\text{-}\Gamma_{\mathbf{0}}\\ \mathcal{I}\mathcal{J}\text{-}\mathrm{Q}_{\mathbf{0}}\end{bmatrix}^{\mathrm{id}})=\min\{|A|:\ A\subseteq\mathcal{D}_{\mathcal{K}},\ (\forall\alpha\in\mathcal{D}_{\mathcal{J}})(\exists\beta\in A)\ \alpha\not\leq_{\mathcal{I}}\beta\}$$

- M. Repický, 2019

$$\mathrm{non}(\begin{bmatrix}\mathcal{K}\text{-}\Gamma_{\mathbf{0}}\\ \mathcal{I}\mathcal{J}\text{-}\mathrm{Q}_{\mathbf{0}}\end{bmatrix})=\min\{|A|:\ A\subseteq\mathcal{D}_{\mathcal{K}},\ (\forall\alpha\in\mathcal{D}_{\mathcal{J}})(\forall\varphi\in\mathcal{D}_{\mathcal{I}})(\exists\beta\in A)\ \alpha\not\leq_{\mathcal{I}}\varphi\circ\beta\}$$

- R. Filipów, A. Kwela and J.Š., 2019

$$\mathrm{non}(\begin{bmatrix}\mathcal{K}\text{-}\Gamma_{\mathbf{0}}\\ \mathcal{I}\text{-}\mathrm{Q}^{\mathrm{s}}_{\mathbf{0}}\end{bmatrix})=\min\{|A|:\ A\subseteq\mathcal{D}_{\mathcal{K}},\ (\forall\varphi\in\mathcal{D}_{\mathcal{I}})(\exists\beta\in A)\ \mathtt{id}\not\leq_{\mathcal{I}}\varphi\circ\beta\}$$

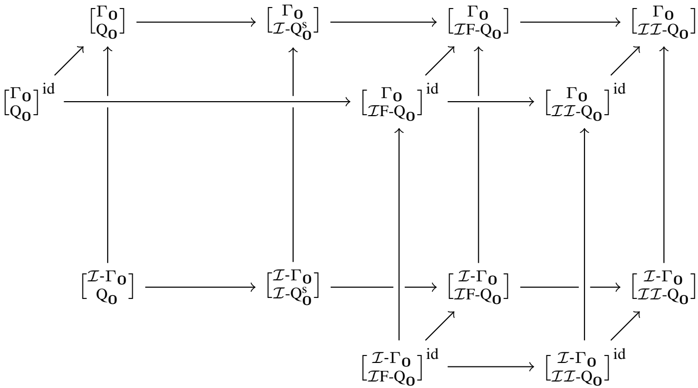
Investigated bounding numbers, several authors

$$\begin{aligned}\mathfrak{b}(\mathcal{I},\mathcal{J},\mathcal{K}) &= \min\{|A| : \ A \subseteq \mathcal{D}_{\mathcal{K}},\ (\forall\alpha\in\mathcal{D}_{\mathcal{J}})(\exists\beta\in A)\ \alpha \not\leq_{\mathcal{I}} \beta\} \\ &= \mathfrak{b}(\mathcal{D}_{\mathcal{K}}, \mathcal{D}_{\mathcal{J}}, \geq_{\mathcal{I}}) \\ &= \mathrm{non}(\begin{bmatrix}\mathcal{K}\text{-}\Gamma_{\mathbf{0}} \\ \mathcal{I}\mathcal{J}\text{-}\mathrm{Q}_{\mathbf{0}}\end{bmatrix}^{\mathrm{id}})\end{aligned}$$

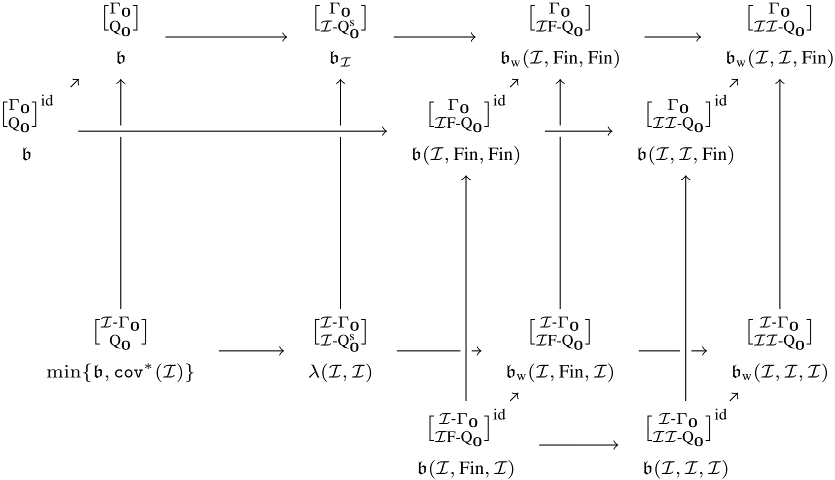
$$\begin{aligned}\mathfrak{b}_{\mathrm{w}}(\mathcal{I},\mathcal{J},\mathcal{K}) &= \min\{|A| : \ A \subseteq \mathcal{D}_{\mathcal{K}},\ (\forall\alpha\in\mathcal{D}_{\mathcal{J}})(\forall\varphi\in\mathcal{D}_{\mathcal{I}})(\exists\beta\in A)\ \alpha \not\leq_{\mathcal{I}} \varphi\circ\beta\} \\ &= \mathfrak{b}(\mathcal{D}_{\mathcal{K}}, \mathcal{D}_{\mathcal{J}}\times\mathcal{D}_{\mathcal{I}}, \mathtt{R}_2\circ\mathtt{L} \geq_{\mathcal{I}} \mathtt{R}_1) \\ &= \mathrm{non}(\begin{bmatrix}\mathcal{K}\text{-}\Gamma_{\mathbf{0}} \\ \mathcal{I}\mathcal{J}\text{-}\mathrm{Q}_{\mathbf{0}}\end{bmatrix})\end{aligned}$$

$$\begin{aligned}\lambda(\mathcal{K},\mathcal{I}) &= \min\{|A| : \ A \subseteq \mathcal{D}_{\mathcal{K}},\ (\forall\varphi\in\mathcal{D}_{\mathcal{I}})(\exists\beta\in A)\ \mathtt{id} \not\leq_{\mathcal{I}} \varphi\circ\beta\} \\ &= \mathfrak{b}(\mathcal{D}_{\mathcal{K}}, \mathcal{D}_{\mathcal{I}}, \mathtt{R}\circ\mathtt{L} \geq_{\mathcal{I}} \mathtt{id}) \\ &= \mathrm{non}(\begin{bmatrix}\mathcal{K}\text{-}\Gamma_{\mathbf{0}} \\ \mathcal{I}\text{-}\mathrm{Q}^{\mathrm{s}}_{\mathbf{0}}\end{bmatrix})\end{aligned}$$

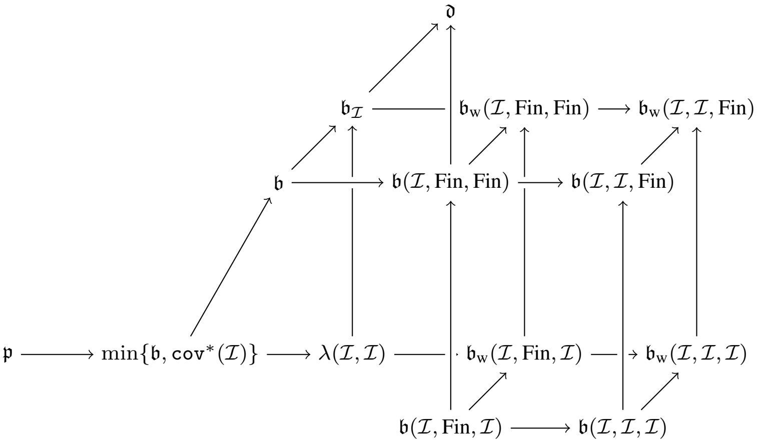
Application



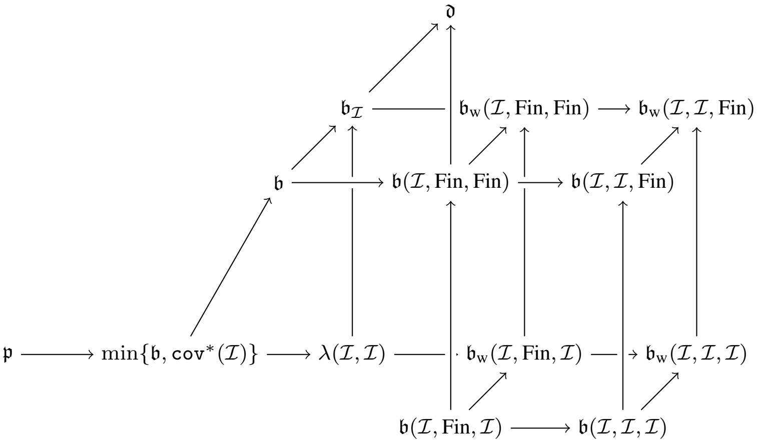
Properties and their critical cardinalities



Critical cardinalities



Critical cardinalities



Proposition

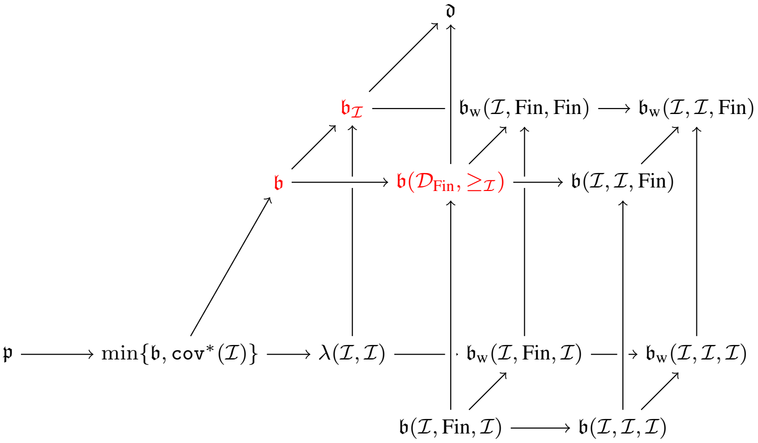
(1) $\min\{\mathrm{cov}^*(\mathcal{I}), \mathfrak{b}_{\mathrm{w}}(\mathcal{I}, \mathrm{Fin}, \mathrm{Fin})\} \leq \mathfrak{b}_{\mathrm{w}}(\mathcal{I}, \mathrm{Fin}, \mathcal{I})$.

(2) $\min\{\mathrm{cov}^*(\mathcal{I}), \mathfrak{b}_{\mathrm{w}}(\mathcal{I}, \mathcal{I}, \mathrm{Fin})\} \leq \mathfrak{b}_{\mathrm{w}}(\mathcal{I}, \mathcal{I}, \mathcal{I})$.

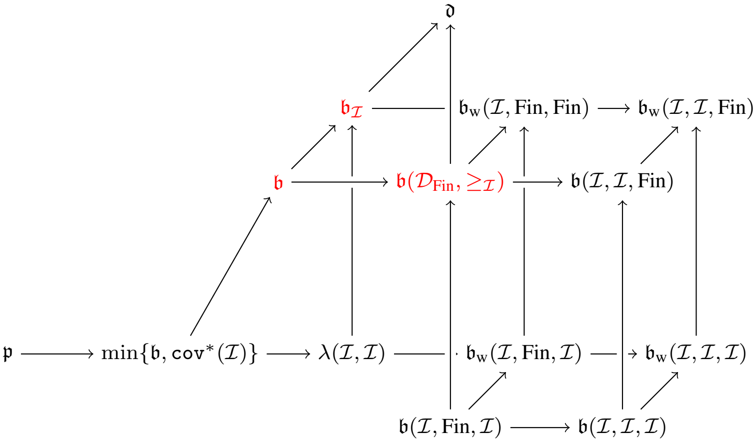
(3) $\min\{\mathrm{cov}^*(\mathcal{I}), \mathfrak{b}_{\mathrm{w}}(\mathcal{I}, \mathcal{I}, \mathrm{Fin})\} \leq \mathfrak{b}$.

(4) $\min\{\mathrm{cov}^*(\mathcal{I}), \mathfrak{b}_{\mathrm{w}}(\mathcal{I}, \mathcal{I}, \mathcal{I})\} \leq \min\{\mathrm{cov}^*(\mathcal{I}), \mathfrak{b}\}$.

Different values 1



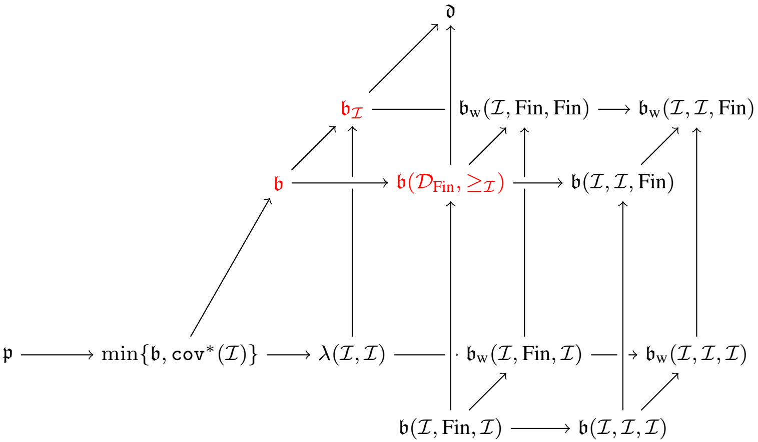
Different values 1



Theorem (M. Canjar 1989)

There is $\mathcal{U}$ such that $\mathfrak{b}_{\mathcal{U}} = \mathfrak{b}(\mathcal{D}_{\mathrm{Fin}}, \geq_{\mathcal{U}}) = \mathtt{cf}(\mathfrak{d})$.

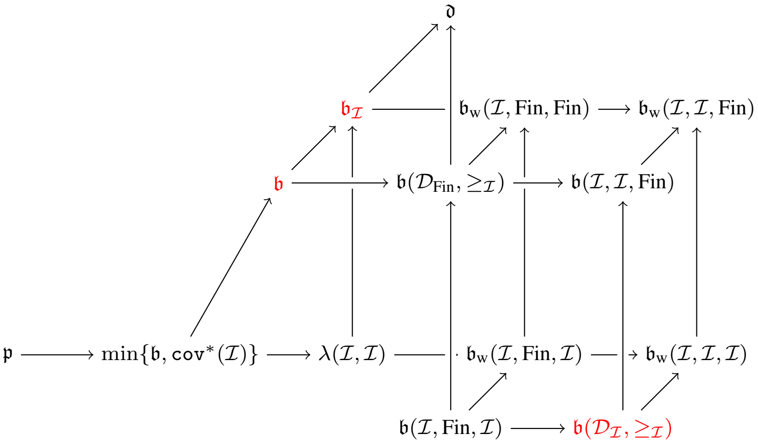
Different values 2



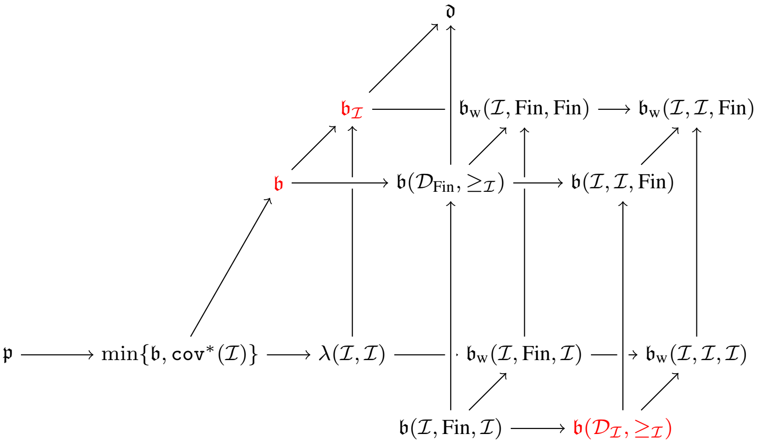
Theorem (M. Canjar 1982, 1988)

In the model obtained by adding λ Cohen reals to a model of **CH** *there exists, for each pair of uncountable regular cardinals $\kappa, \kappa' \leq \lambda$, an ultrafilter $\mathcal{U}$ such that $\mathfrak{b}_{\mathcal{U}} = \kappa$ and $\mathfrak{b}(\mathcal{D}_{\mathrm{Fin}}, \geq_{\mathcal{U}}) = \kappa'$.*

Different values 3



Different values 3



Theorem (M. Canjar 1988)

In the model obtained by adding λ Cohen reals to a model of **CH** *there exists, for each pair of uncountable regular cardinals $\kappa, \kappa' \leq \lambda$, an ultrafilter $\mathcal{U}$ such that $\mathfrak{b}_{\mathcal{U}} = \kappa$ and $\mathfrak{b}(\mathcal{D}_{\mathcal{U}}, \geq_{\mathcal{U}}) = \kappa'$.*

Sample values

R. Filipów and A. Kwela, 2019

$\mathcal{I}$	$\mathfrak{b}(\mathcal{I},\mathrm{Fin},\mathcal{I})$	$\mathfrak{b}(\mathcal{I},\mathcal{I},\mathcal{I})$	$\mathfrak{b}(\mathcal{I},\mathrm{Fin},\mathrm{Fin})$	$\mathfrak{b}(\mathcal{I},\mathcal{I},\mathrm{Fin})$
$\mathrm{Fin}\times\mathrm{Fin}$	1	$\mathfrak{b}$	$\mathfrak{b}$	$+\infty$
$\mathcal{E}D$	1	$\aleph_1$	$\mathfrak{b}$	$\mathfrak{b}$
$\mathtt{conv}$	1	$\aleph_1$	$\mathfrak{b}$	?
$\mathcal{Z}$	$\mathfrak{b}$	$\mathfrak{b}$	$\mathfrak{b}$	$\mathfrak{b}$

V. Šottová and J.Š., 2019

$\mathcal{I}$	$\mathtt{cov}^*(\mathcal{I})$	$\lambda(\mathcal{I},\mathrm{Fin})$	$\lambda(\mathcal{I},\mathcal{I})$
$\mathrm{Fin}\times\mathrm{Fin}$	$\mathfrak{b}$	$\mathfrak{b}$	$\mathfrak{b}$
$\mathcal{E}D$	$\mathtt{non}(\mathcal{M})$	$\mathfrak{b}$	$\mathfrak{b}$
$\mathtt{conv}$	$\mathfrak{c}$	$\mathfrak{b}$	$\mathfrak{b}$
$\mathtt{nwd}$	$\mathtt{cov}(\mathcal{M})$	$\mathtt{add}(\mathcal{M})$	$\mathtt{add}(\mathcal{M})\Box\mathfrak{b}$
$\mathtt{cov}^*(\mathcal{I})=\mathfrak{p}$	$\mathfrak{p}$	$\mathfrak{p}$	$\mathfrak{p}\Box\mathfrak{b}_{\mathcal{I}}$

Bukovský L., Das P. and Šupina J.: *Ideal quasi-normal convergence and related notions*, Colloq. Math. **146** (2017), 265–281.

Canjar M.: *Model-theoretic properties of countable ultraproducts without the Continuum Hypothesis*, ProQuest LLC, Ann Arbor, MI, 1982, Thesis (Ph.D.) - University of Michigan.

Canjar M.: *Countable ultraproducts without CH**, Ann. Pure Appl. Logic **37** (1988), 1–79.

Canjar M.: *Cofinalities of countable ultraproducts: the existence theorem*, Notre Dame J. Formal Logic **30** (1989), 539–542.

Das P. and Chandra D., *Spaces not distinguishing pointwise and $\mathcal{I}$-quasinormal convergence of real functions*, Comment. Math. Univ. Carolin. **54** (2013), 83–96.

Filipów R. and Kwela A., *Yet another ideal version of the bounding number*, manuscript.

Filipów R., Kwela A. and Šupina J., *Different ideal versions of wQN-space and QN-space*, manuscript.

Filipów R. and Staniszewski M., *On ideal equal convergence*, Cent. Eur. J. Math **12** (2014), 896–910.

Filipów R. and Staniszewski M., *Pointwise versus equal (quasi-normal) convergence via ideals*, J. Math. Anal. Appl. **422** (2015), 995–1006.

Filipów R. and Szuca P., *Three kinds of convergence and the associated $\mathcal{I}$-Baire classes*, J. Math. Anal. Appl. **391** (2012), 1–9.

Hrušák M., *Combinatorics of filters and ideals,* Contemp. Math. **533** (2011), 29–69.

Kwela A., *Ideal weak QN-space*, Topology Appl. **240** (2018), 98–115.

Repický M., *Spaces not distinguishing ideal convergences of real-valued functions*, preprint.

Šottová V. and Šupina J.: *Principle S1(P;R): ideals and functions*, Topology Appl. **258** (2019), 282–304.

Staniszewski M.: *On ideal equal convergence II*, J. Math. Anal. Appl. **451** (2017), 1179–1197.

Šupina J.: *Ideal* QN*-spaces*, J. Math. Anal. Appl. **434** (2016) 477–491.

Thanks for Your attention!

Ideal versions of wQN-space and QN-space

Let X be a topological space.

$\mathcal{K}$-$\Gamma_{\mathbf{0}}$, $\mathcal{I}\mathcal{J}$-$\mathrm{Q}_{\mathbf{0}}$, $\mathcal{I}$-$\mathrm{Q}^{\mathrm{s}}_{\mathbf{0}}$, is the family of all $\mathcal{K}$-convergent sequences of continuous functions to $\mathbf{0}$, $\mathcal{I}\mathcal{J}$QN-convergent sequences of continuous functions to $\mathbf{0}$ and $\mathrm{s}\mathcal{I}$QN-convergent sequences of continuous functions to $\mathbf{0}$, respectively.

$\mathcal{R}$ is $\mathcal{I}\mathcal{J}$-$\mathrm{Q}_{\mathbf{0}}$ or $\mathcal{I}$-$\mathrm{Q}^{\mathrm{s}}_{\mathbf{0}}$

- $\mathrm{C}_p(X)$ is a $[\mathcal{K}\text{-}\Gamma_{\mathbf{0}}, \mathcal{R}]$-space if for every $\langle f_n : n \in \omega \rangle \in \mathcal{K}$-$\Gamma_{\mathbf{0}}$ there is $\langle n_m : m \in \omega \rangle$ such that $\langle f_{n_m} : m \in \omega \rangle \in \mathcal{R}$.

$\mathcal{J} \subseteq \mathcal{I}$

- $\mathrm{C}_p(X)$ is a $[\mathcal{K}\text{-}\Gamma_{\mathbf{0}}, \mathcal{I}\mathcal{J}\text{-}\mathrm{Q}_{\mathbf{0}}]^{\mathrm{id}}$-space if $\mathcal{K}$-$\Gamma_{\mathbf{0}} \subseteq \mathcal{I}\mathcal{J}$-$\mathrm{Q}_{\mathbf{0}}$.