Admissibility of Cut for Sequent Calculus related to n-labelled Tableaux

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VERIFICATIONIST versus FALSIFICATIONIST APPROACH TO SEQUENT CALCULUS

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VA $\Longrightarrow$ Schütte-style SC (or Rasiowa dual tableaux).

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whereas in FA it is just the opposite.

Verificationist/Falsificationist Version:

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VA: Sequents of the form $\Gamma \mid \Delta \Rightarrow \Sigma$ which are satisfied in a L3-matrix iff some $\varphi \in \Gamma$ is 0, or some $\psi \in \Delta$ is 1/2, or some $\chi \in \Sigma$ is 1, under some h.

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FA: Sequents of the form $\Gamma \Rightarrow \Delta \mid \Sigma$ which are falsified in a L3-matrix iff all $\varphi \in \Gamma$ are 1 and all $\psi \in \Delta$ are 1/2, and all $\chi \in \Sigma$ are 0, under some h.

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FA: A sequent is counted as an axiom if either $\Gamma \cap \Delta$ or $\Gamma \cap \Sigma$ or $\Delta \cap \Sigma$ is nonempty (or of the form: $\varphi \Rightarrow \varphi \mid$, $\varphi \Rightarrow \mid \varphi$, $\Rightarrow \varphi \mid \varphi$)

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$(\neg \mid\Rightarrow)\ \dfrac{\Gamma \mid \Delta \Rightarrow \Sigma, \varphi}{\neg\varphi, \Gamma \mid \Delta \Rightarrow \Sigma}$ $\quad (\mid \neg \Rightarrow)\ \dfrac{\Gamma \mid \varphi, \Delta \Rightarrow \Sigma}{\Gamma \mid \neg\varphi, \Delta \Rightarrow \Sigma}$

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$$(\Rightarrow\to)\ \frac{\varphi, \Gamma \mid \varphi, \Delta \Rightarrow \Sigma, \psi \quad \varphi, \Gamma \mid \psi, \Delta \Rightarrow \Sigma, \psi}{\Gamma \mid \Delta \Rightarrow \Sigma, \varphi \to \psi}$$ versus

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$$(\rightarrow\Rightarrow)\ \frac{\Sigma \Rightarrow \Delta \mid \Gamma, \varphi \quad \Sigma \Rightarrow \Delta, \varphi, \psi \mid \Gamma \quad \psi, \Sigma \Rightarrow \Delta \mid \Gamma}{\varphi \rightarrow \psi, \Gamma \Rightarrow \Delta \mid \Sigma}$$

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Derivability:

FA: $\Gamma \vdash \varphi$ iff there is a proof of $\Gamma \Rightarrow \varphi \mid$ and $\Gamma \Rightarrow\mid \varphi$
but!

VA: $\Gamma \vdash \varphi$ iff there is a proof of $\Pi(\Gamma) \Rightarrow \varphi$
where $\Pi(\Gamma)$ is one of the $2^{|\Gamma|}$ quasi-partitions of Γ.

Verificationist/Falsificationist Version – Cut:

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$$\frac{\Gamma \mid \Delta, \varphi \Rightarrow \Sigma \quad \varphi, \Gamma \mid \Delta \Rightarrow \Sigma}{\Gamma \mid \Delta \Rightarrow \Sigma}$$ versus

$$(3-cut) \quad \frac{\Sigma \Rightarrow \Delta \mid \Gamma, \varphi \quad \Sigma \Rightarrow \Delta, \varphi \mid \Gamma \quad \varphi, \Sigma \Rightarrow \Delta \mid \Gamma}{\Gamma \Rightarrow \Delta \mid \Sigma}$$

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For VA cut admissibility proved by Baaz, Fermüller and Zach [1994].

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2. h-p admissibility of Weakening (three versions);
3. h-p invertibility of all rules (with respect to all premisses);
4. h-p invertibility of Contraction (in three versions).

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$$(\to\Rightarrow)\ \dfrac{\Sigma \Rightarrow \Delta, \varphi \mid \Gamma, \psi, \varphi \qquad \Sigma \Rightarrow \Delta, \varphi, \varphi, \psi \mid \Gamma, \psi \qquad \psi, \Sigma \Rightarrow \Delta, \varphi \mid \Gamma, \psi}{\varphi \to \psi, \Sigma \Rightarrow \Delta, \varphi \mid \Gamma, \psi}$$

$$(\Rightarrow\to\mid)\ \dfrac{\varphi \to \psi, \Sigma \Rightarrow \Delta, \varphi \mid \Gamma, \psi \qquad \varphi \to \psi, \varphi, \Sigma \Rightarrow \Delta, \psi}{\varphi \to \psi, \Sigma \Rightarrow \Delta, \varphi \to \psi \mid \Gamma}$$

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the same must be done for $\varphi \to \psi, \Sigma \Rightarrow \Delta \mid \varphi \to \psi, \Gamma$ and for $\Sigma \Rightarrow \Delta, \varphi \to \psi \mid \Gamma, \varphi \to \psi$.

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Dragalin-style proof – three main cases:

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Cut Admissibility for Falsificationist Version – principal cut-formulae:

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Cut Admissibility for Falsificationist Version – principal cut-formulae:

$$\dfrac{\Sigma \Rightarrow \Delta, \varphi \wedge \psi \mid \Gamma \qquad (\Rightarrow\mid \wedge)\ \dfrac{\Pi \Rightarrow \Lambda \mid \Theta, \varphi \qquad \Pi \Rightarrow \Lambda \mid \Theta, \psi}{\Pi \Rightarrow \Lambda \mid \Theta, \varphi \wedge \psi} \qquad \dfrac{\varphi, \psi, \Xi \Rightarrow \Upsilon \mid \Omega}{\varphi \wedge \psi, \Xi \Rightarrow \Upsilon \mid \Omega}\ (\wedge \Rightarrow)}{\Sigma, \Pi, \Xi \Rightarrow \Delta, \Lambda, \Upsilon \mid \Gamma, \Theta, \Omega}\ (3 - Cut)$$

where the leftmost premiss is deduced by:

$$(\Rightarrow \wedge \mid)\ \dfrac{\varphi, \Sigma \Rightarrow \Delta, \psi \mid \Gamma \qquad \Sigma \Rightarrow \Delta, \varphi, \psi \mid \Gamma \qquad \psi, \Sigma \Rightarrow \Delta, \varphi \mid \Gamma}{\Sigma \Rightarrow \Delta, \varphi \wedge \psi \mid \Gamma}$$

Cut Admissibility for Falsificationist Version – principal cut-formulae:

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Cut Admissibility for Falsificationist Version – principal cut-formulae:

$$\dfrac{\dfrac{P \qquad \Pi \Rightarrow \Lambda \mid \Theta, \varphi \qquad \dfrac{\Pi \Rightarrow \Lambda \mid \Theta, \psi \qquad \varphi, \psi, \Xi \Rightarrow \Upsilon \mid \Omega \qquad \varphi, \Sigma \Rightarrow \Delta, \psi \mid \Gamma}{\varphi, \Sigma, \Pi, \Xi \Rightarrow \Delta, \Lambda, \Upsilon \mid \Gamma, \Theta, \Omega}\,(3-Cut)}{\Sigma, \Sigma, \Pi, \Pi, \Xi \Rightarrow \Delta, \Delta, \Lambda, \Lambda, \Upsilon \mid \Gamma, \Gamma, \Theta, \Theta, \Omega}\,(3-Cut)}{\Sigma, \Pi, \Xi \Rightarrow \Delta, \Lambda, \Upsilon \mid \Gamma, \Theta, \Omega}\,(C)$$

where the leftmost premiss P is:

$$(3-Cut)\,\dfrac{\Pi \Rightarrow \Lambda \mid \Theta, \psi \qquad \psi, \Sigma \Rightarrow \Delta, \varphi \mid \Gamma \qquad \Sigma \Rightarrow \Delta, \varphi, \psi \mid \Gamma}{(C)\,\dfrac{\Sigma, \Sigma, \Pi \Rightarrow \Delta, \Delta, \Lambda, \varphi \mid \Gamma, \Gamma, \Theta}{\Sigma, \Pi \Rightarrow \Delta, \Lambda, \varphi \mid \Gamma, \Theta}}$$