Logic Colloquium 2019

book of abstracts

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D. Chodounský, Š. Stejskalová, J. Verner (eds.)

Association for Symbolic Logic

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Daniel T. Soukup *Faculty of Mathematics, University of Vienna*
Jan Starý *Czech Technical University*
Sarka Stejskalova *Department of Logic, Charles University / Institute of Mathematics, Czech Academy of Sciences*
Bernhard Stoinski *Institut für Dynamsiche Logik*
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1 | Plenary talks

Samson Abramsky

Relating Structure and Power: a junction between categorical semantics, model theory and descriptive complexity

There is a remarkable divide in the field of logic in Computer Science, between two distinct strands: one focussing on semantics and compositionality ("Structure"), the other on expressiveness and complexity ("Power"). It is remarkable because these two fundamental aspects are studied using almost disjoint technical languages and methods, by almost disjoint research communities. We believe that bridging this divide is a major issue in Computer Science, and may hold the key to fundamental advances in the field.

In this talk, we describe a novel approach to relating categorical semantics, which exemplifies the first strand, to finite model theory, which exemplifies the second. It is based on [1, 2], and ongoing joint work with Nihil Shah, Tom Paine and Anuj Dawar.

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Zoé Chatzidakis

Notions of difference closures of difference fields

It is well known that a differential field K of characteristic 0 is contained in a differential field which is differentially closed and has the property that it K-embeds in every differentially closed field containing K. Such a field is called a differential closure of K, and it is unique up to K-isomorphism. In other words, prime models exist and are unique. The proof uses the fact that the theory of differentially closed fields of characteristic 0 is totally transcendental.

One can ask the same question about difference fields: do they have a difference closure, and is it unique? The immediate answer to both these questions is no, for trivial reasons: in most cases, there are continuum many ways of extending an automorphism of a field to its algebraic closure. Therefore a natural requirement is to impose that the field K be algebraically closed. Similarly, if the subfield of K fixed by the automorphism is not pseudo-finite, then there are continuum many ways of extending it to a pseudo-finite field, so one needs to add the hypothesis that the fixed subfield of K is pseudo-finite.

In this talk I will show by an example that even these two conditions do not suffice.

There are two (and more) natural strengthenings of the notion of difference closure, and we show that in characteristic 0, these notions do admit unique prime models over any algebraically closed difference field K, provided the subfield of K fixed by the automorphism is large enough.

In model-theoretic terms, this corresponds to the existence and uniqueness of a-prime or κ-prime models.

In characteristic $p > 0$, no such result can hold.

Osvaldo Guzman

The ultrafilter and almostdisjointness numbers

The *cardinal invariants of the continuum* are certain uncountable cardinals that are less or equal to the cardinality of the real numbers. This relation and nonrelation between this cardinals has been deeply studied by set theorists. In this talk, we will focus on the following two invariants: The *ultrafilter number* $\mathfrak{u}$, which is defined as the smallest size of a base of an ultrafilter, and the *almost disjointness number* $\mathfrak{a}$, which is the smallest size of a MAD family. The consistency of the inequality $\mathfrak{a} < \mathfrak{u}$ is well known and easy to prove. The consistency of the inequality $\mathfrak{u} < \mathfrak{a}$ is much harder to obtain. It was Shelah who proved that, under the assumption that there is a measurable cardinal, there is model of $\omega_1 < \mathfrak{u} < \mathfrak{a}$. In spite of the beauty of the result, the following questions remained open:

(Shelah) Does *CON(ZFC)* implies *CON(ZFC +* $\mathfrak{u} < \mathfrak{a}$*)?*

(Brendle) Is it consistent that $\omega_1 = \mathfrak{u} < \mathfrak{a}$?

In this talk, we are going to see how to provide a positive answer to both questions. This is joint work with Damjan Kalajdzievski. No previous knowledge of cardinal invariants of the continuum is needed for the talk.

Matthew Harrison-Trainor

Describing countable structures

Given a countable structure, how do we measure its complexity? One way to do this is by measuring the complexity of describing that structure. Dana Scott proved that for each countable structure $\mathcal{A}$ there is a sentence of infinitary logic that is true of $\mathcal{A}$ and not true of any other countable structure. We can think of such a sentence as a description of the structure, and call any such sentence a Scott sentence. The Scott complexity of a structure is the complexity of the simplest Scott sentence for that structure. The Scott complexity of a structure is tightly related to other notions of complexity, such as the complexity of understanding automorphisms of the structure, or of finding isomorphisms between different copies of the structure. This talk will begin with a general overview of the area followed by a number of recent results on finitely generated structures and on structures of high Scott rank.

Ulrich Kohlenbach

Local proof-theoretic foundations, proof-theoretic tameness and proof mining

Recently, John Baldwin pointed to a ‘paradigm shift in model theory’ stressing that while early 20th century logic focused on the formalization of all of mathematics, model theory increasingly studied specific areas of mathematics (local formalizations) with an emphasis on tame structures ([1]). We will argue that also the successful use of proof-theoretic methods in core mathematics (‘proof mining’, [2]) in recent decades was made possible by developing logical metatheorems tailored for applications to particular classes of theorems and proofs in specific areas of mathematics. In analysis, these classes of theorems (e.g. convergence statements), however, do involve arithmetic (together with analytical and geometric structures) and so are not tame in the model-theoretic sense but could in principle display Gödelian or huge growth phenomena. It is an empirical fact, though, that with a few notable exceptions (which still are primitive recursive in the sense of Gödel’s T), proofs in existing ordinary analysis are largely tame in the sense of allowing for the extraction of bounds of rather low complexity. To determine the amount of ‘proof-theoretic tameness’ in a given proof requires a proof-theoretic analysis in each case. We will discuss two recent applications of proof mining, one of which displays a highly tame (polynomial) behavior ([3]) whereas the other one as it stands uses primitive recursion of type-1 level ([4]).

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Jan Krajíček

Model theory and proof complexity

Mathematical logic and computational complexity theory have many topics in common. In most cases the links between the two fields are fostered by finite combinatorics, manifesting either via proof theory or via finite model theory.

There are, however, also topics in complexity theory where infinitary methods of logic shed a new light on old problems. I will discuss, in particular, how non-finite model theory relates to proof complexity. The relevant model theoretical problems involve constructions of models of bounded arithmetic and of expanded extensions of pseudo-finite structures. I will describe forcing with random variables aimed at tackling these problems, and give some examples of results that can be obtained in this way.

Hannes Leitgeb

Ramsification and Semantic Indeterminacy

Since the publication of Ramsey's (1929) "Theories", the *Ramsification* of scientific theories has become a major tool in scientific theory interpretation and reconstruction. In this talk, I will argue that the Ramsification of classical Tarskian semantics can also help us overcome problems that result from the vagueness of ordinary terms in natural language or from the theoreticity and open-endedness of technical terms in mathematical and scientific language. The resulting "Ramsey semantics" saves all of classical logic and almost all of classical semantics, while embracing semantic indeterminacy without going down an epistemicist or supervaluationist road. In application to the semantics of the language(s) of mathematics, it goes some way towards a reconciliation between classical mathematics and intuitionistic concerns.

Dilip Raghavan

Higher cardinal invariants.

There has been a recent resurgence of research into cardinal invariants at regular uncountable cardinals. This recent work has revealed many differences between cardinal invariants at ω and their analogues at uncountable cardinals. One unexpected conclusion is that there seem to be more ZFC inequalities provable at uncountable cardinals than at ω. The study of cardinal invariants at uncountable cardinals has also led to the development of novel forcing techniques, mostly notably the method of forcing with Boolean ultrapowers, which was introduced in [4] to investigate the higher analogue of the almost disjointness number.

I will present a survey of some of this recent work, restricting my attention to six combinatorial cardinal characteristics at regular uncountable cardinals. Some ZFC results, such as the ones in [1] and [2], as well as some consistency results, such as the ones in [3], will be mentioned. Time permitting, I will expose the method of Boolean ultrapowers as developed in [4] and sketch some of the consistency results at regular uncountable cardinals that can be obtained using this method.

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Michael Rathjen

Well-ordering principles in proof theory and reverse mathematics

Several results about the equivalence of familiar theories of reverse mathematics with certain well-ordering principles have been proved (Friedman, Marcone, Montalban et al.) by recursion-theoretic and combinatorial methods and also by proof theory (Afshari, Girard, R, Weiermann et al.), employing deduction search trees and cut elimination theorems in infinitary logics with ordinal bounds.

One goal of the talks is to present a general methodology underlying these results which in many cases allows one to establish an equivalence between two types of statements. The first type is concerned with the existence of ω-models of a theory whereas the second type asserts that a certain (usually well-known) elementary operation on orderings preserves the property of being well-ordered. These operations are related to ordinal representation systems (ors) that play a central role in proof theory. The question of naturality of ors has vexed logicians for a long time. While ors have a low computational complexity, their "true" nature evades characterization in those terms. One attempt has been to describe their structural properties in category-theoretic terms (Aczel, Feferman, Girard et al.). Some of these ideas will be discussed in the talks.

A second goal is to present rather recent developments (due to Arai, Freund, R), especially work by Freund on higher order well-ordering principles and comprehension.

Vincenzo de Risi

Drawing Lines through Rivers and Cities.The Meaning of Postulates from Euclid to Hilbert

The talk attempts to sketch a history of the development of the meaning of mathematical principles from Antiquity to the Modern Age. Euclid's own conception of principles (definitions, postulates, common notions) was widely different from ours, and it requires some exercise to understand what did it mean for him to ground geometry on a set of principles. We will explore how Euclid's own views on the foundations of mathematics were interpreted and misinterpreted in Late Antiquity, and how a new conception of principles arose in medieval Scholasticism. Such interpretation of axioms and postulates, that stemmed in the commentaries to Aristotle's Analytics, was immensely influent in the early modern age, and was endorsed, with various degrees of variance, by authors such as Clavius, Wallis, Leibniz or Euler. In the 18th Century, on the other hand, a new conception of axiom began to rise in the works of Lambert and Bolzano. This last development in the meaning of a mathematical principle paved the way for some of the modern understandings of it, in the works of Frege, Hilbert and others. The talk will also present a survey of the main axioms employed in the modern age to ground elementary geometry, which greatly differed from Euclid's original principles and were later collected in the books of the foundations of geometry by Peano, Pasch and Hilbert.

Gil Sagi

Logic and natural language: commitments and constraints

Most of the contemporary research in logic is carried out with respect to formal languages. Logic, however, is said to be concerned with correct reasoning, and it is natural language that we usually reason in. Thus, in order to assess the validity of arguments in natural language, it is useful to formalize them: to provide matching arguments in a formal language where logical properties become perspicuous. It has been recognized in the literature that formalization is far from a trivial process. One must discern the logical from the nonlogical in the sentence, a process that requires theorizing that goes beyond the mere understanding of the sentence formalized [1]. Moreover, according to some, "logical forms are not to be discovered but rather established and ascribed to expressions within processes of the reflective equilibrium" [2]. I concur. I argue that logical forms are imposed, and that furthermore, they carry a normative force in the form of commitments on behalf of the theorizer.

In previous work [3], I proposed a model-theoretic framework of "semantic constraints", where there is no strict distinction between logical and nonlogical vocabulary. The form of sentences in a formal language is determined rather by a set of constraints on models. In the present paper, I show how this framework can also be used in the process of formalization, where the semantic constraints are conceived of as commitments made with respect to the language.

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Thomas Scanlon

Over six decades of the model theory of valued fields

Inspired by Angus Macintyre's lecture "Twenty years of p-adic model theory" at the Logic Colloquium '84 in Manchester, I widen the scope

exploring the role that the theory of valued fields hasplayed (and continues to play) in the internal development of model theory and in the applications of model theory to other parts of mathematics.

Rineke Verbrugge

Zero-one laws for provability logic and some of its siblings

Glebskii and colleagues proved in the late 1960s that each formula of first-order logic without constants and function symbols obeys a zero-one law. That is, every such formula is either almost surely valid or almost surely not valid: As the number of elements of finite models increases, each formula holds either in almost all or in almost no models of that size. As a consequence, many properties of models, such as having an even number of elements, cannot be expressed in the language of first-order logic without constants and function symbols. In a 1994 paper, Halpern and Kapron proved similar zero-one laws for classes of models corresponding to the modal logics K, T, S4, and S5.

In this presentation, we discuss zero-one laws for some modal logics that impose structural restrictions on their models; all three logics that we are interested in are sound and complete with respect to finite partial orders, with different extra restrictions per logic. We prove zero-one laws for provability logic and its two siblings Grzegorczyk logic and weak Grzegorczyk logic, with respect to model validity. Moreover, for all three logics, we axiomatize validity in almost all relevant finite models, leading to three different axiom systems. In the proofs, we use a combinatorial result by Kleitman and Rothschild about the structure of almost all finite partial orders. We also discuss the question whether for the three sibling logics, validity in almost all relevant finite frames can be axiomatized as well. Finally, we consider the complexity of deciding whether a given formula is almost surely valid in the relevant finite models.

Martin Ziegler

Logic of Computing with Continuous Data: Foundations of Numerical Software Engineering

Over 30 years after introducing the IEEE 754 standard, Numerics still gyrates around floating point numbers: from specification (e.g. of `e04bbc` in the NAG library) via analysis (unit-cost/realRAM/Blum-Shub-Smale model) and implementation to verification. Yet their violation of Distributive Law, of Intermediate-Value Theorem, and of Quantifier Elimination hampers rigorous approaches to Numerical Software Engineering: Modern Calculus builds on real (rather than rational) numbers for a reason!

We reconcile the convenient algebraic perspective on real computation (Bürgisser) with Computable Analysis (Grzegorczyk, Pour-El, Weihrauch) by developing Turing-complete semantics for operating on continuous structures (Poizat, Zucker). This imperative counterpart to realPCF (Escardo) extends the powerful formal tools of Software Engineering from the discrete to the continuous realm with benefits to numerical practice.

2 | Set Theory

David Chodounský
Osvaldo Guzmán

Yair Hayut

Stationary Reflection at the successor of a singular cardinal

In the paper [2], the consistency of stationary reflection holds at all stationary subset of $\aleph_{\omega+1}$ which concentrate on ordinals of uncountable cofinality, was obtained from the existence of a cardinal κ which is κ^+-supercompact. Using a similar method, Zeman showed in [5] that $\neg\square_{\aleph_\omega}$ is consistent relative to the weaker assumption — a measurable subcompact cardinal. In both cases, Prikry forcing is used in order to singularize a measurable cardinal that will become the new $\aleph_\omega$. When trying to improve those results in order to obtain full stationary reflection at $\aleph_{\omega+1}$ one needs to deal with the non-reflecting stationary sets which are introduced by the Prikry forcing.

In this talk I will describe the main ideas behind the method which is used in a joint work with Spencer Unger, [4]. In this work we obtain full stationary reflection at $\aleph_{\omega+1}$, starting from a large cardinal axiom weaker than the one from [2]. This method uses the ideas of [1] and [3], and enables us to analyse the properties of a Prikry type generic extensions by using internal analysis of some iterated ultrapowers, as well as construct a specialized Prikry type forcing notion with a controlled behaviour for our problem.

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Heike Mildenberger*, Saharon Shelah

Generalised Miller forcing may collapse cardinals

We show that it is independent whether club-κ-Miller forcing preserves κ^{++}. With club guessing and other prediction principles we show that under $\kappa^{<\kappa} > \kappa$, club-$\kappa$-Miller forcing collapses $\kappa^{<\kappa}$ to κ. We investigate variants of κ-Miller forcing and draw connections to the forcing $([\kappa]^{\kappa}, \subseteq)$.

Daniel T. Soukup

Through the lense of uniformization

The main goal of this talk is to review recent applications of the uniformization property of ladder systems on ω_1. This notion played a critical role in S. Shelah's solution of the Whitehead problem; in the understanding of forcing axioms which can be consistent with CH [2]; and in J. Moore's work on minimal uncountable linear orders [1]. We shall focus on more recent results concerning edge colourings of graphs with uncountable chromatic number (joint work with M. Dzamonja, T. Inamdar and J. Steprans) and questions about minimal uncountable linear orders [5, 6]. The latter topic leads to the analysis of uniformizations on Aronszajn trees [3, 4] which we shall touch on briefly.

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Andy Zucker

Bernoulli disjointness

We consider the concept of disjointness for topological dynamical systems, introduced by Furstenberg. We show that for every discrete group, every minimal flow is disjoint from the Bernoulli shift. We apply this to give a negative answer to the "Ellis problem" for all such groups. For countable groups, we show in addition that there exists a continuum-sized family of mutually disjoint free minimal systems. Using this, we can identify the underlying space of the universal minimal flow of every countable group, generalizing results of Balcar-Błaszczyk and Turek. In the course of the proof, we also show that every countable ICC group admits a free minimal proximal flow, answering a question of Frisch, Tamuz, and Vahidi Ferdowsi. This is joint work with Eli Glasner, Todor Tsankov, and Benjamin Weiss.

Frode A. Bjørdal

Capture, Replacement, Specification

Let **W** be Zermelo set theory **Z** minus specification and choice. For $\alpha(v,x,y)$ any first order condition in the language of set theory on the indicated free variables, legislate:

Axiom of Capture: $\forall v \exists w \forall x (x \in w \leftrightarrow \exists y (y \in v \wedge \alpha(v,x,y) \wedge (\forall z)(\alpha(v,z,y) \rightarrow x = z)))$

Let **ZF** be Zermelo-Fraenkel set theory: We show **ZF** $=$ **W**$+$ **Axiom of Capture**.

Capture avoids the cumbersome restriction to *functional* condition, and is justified by the idea that we should accept as many instances of naive comprehension as possible. Versions of capture are of use in the context of the author's alternative set theory £ as in [1] because they allow for more flexibility in expressing useful closure principles.

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Evgeny Gordon

On extension of Haar measure in σ-compact groups

In the paper [1] the model of ZFC, where every set of reals, definable by a sequence of ordinals is Lebesgue measurable was constructed under assumptions of existence of an inaccessible cardinal. On the base of this model the model of ZF+DC, in which every set of reals is Lebesgue measurable was presented. In [2] it was proved without the assumption of existence of inaccessible cardinal that the possibility to extend the Lebesgue measure to a non-regular σ-additive invariant measure defined on all sets of reals is consistent with ZF+DC. Later on Shelah proved that the assumption of existence of inaccessible cardinal cannot be removed from the Solovay's result [3]. In the talk we present the following theorem.

Theorem. *Let α be an arbitrary ordinal definable in ZF. Denote $Base(X,\beta)$ and $Ext(X,\beta)$ the statements*

1. *"X is a σ-compact group with the base of topology of cardinality β";*
2. *"In a σ-compact group X the left Haar measure can be extended to a left invariant σ-additive measure defined on all subsets of X definable by a β-sequence of ordinals".*

respectively. Then the following proposition is consistent with ZFC:

$$\forall X \forall \beta < \aleph_\alpha < |\mathbb{R}| \ (Base(X,\beta) \longrightarrow Ext(X,\beta))$$

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John Howe

Ramsey degrees of structures with equivalence relations

The Ramsey theory of homogeneous structures is an attempt to answer the question of how the infinite version Ramsey's theorem changes if instead of having ω as a pure set, we require more structure. This dates back to work of Galvin and Devlin on the rationals, with more recent results about graphs coming Sauer and Dobrinen. Both of these have used techniques involving tree Ramsey theorems, whereas Nguyen Van Thé's work about ultrametric spaces uses the classical Ramsey theorem. I will explain some recent work unifying these approaches and yielding results about the generic ordered equivalence relation.

Joanna Jureczko

New results on partitioner-representable algebras

In 1982 Baumgartner and Weese introduced the natural notion of partitioners.

Remind that if F is a mad family, then a set $a \subseteq \omega$ is called a *paritioner* of F iff for all $b \in F$ either $b - a$ or $b \cap a$ is finite. Then, if $\mathbb{B}$ is a Boolean algebra and I is an ideal in $\mathbb{B}$ generated by F and finite sets, the algebra $\mathbb{B}/I$ is called the *partition algebra* of F. If a Boolean algebra is isomorphic to the partition algebra of some mad family, then such an algebra is called to be *representable*.

In [1] the authors proved several important theorems in this subject, among others they showed that under (CH) each Boolean algebra of cardinality $\leqslant \mathfrak{c}$ is representable. They also show that there are algebras which are non-representable in some models.

The authors in [1] also posed a number of problems which were solved later, (see [3]). We will show the solution of Problem 3: Must every representable algebra be embeddable in $P(\omega)/fin$? Among others we will show that there are some models in which $P(\omega_1)$ is embeddable in $P(\omega)/fin$ but not representable, and conversely. The most our unexpected result is that there is a model in which $P(\omega)/fin$ is not representable. During the talk we also present some related results.
This is the joint work with Ryszard Frankiewicz.

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Borisa Kuzeljević

Antichains of copies of ultrahomogeneous structures

We analyze possible cardinalities of maximal antichains of isomorphic copies of countable ultrahomogeneous structures. For a countable ultrahomogeneous relational structure X, $\mathbb{P}(X)$ denotes the set of all substructures of X isomorphic to it. A copy $Y \in \mathbb{P}(X)$ is called *large* if it intersects each orbit of X. We say that a collection $\mathcal{A}$ of copies of X is an antichain in $\mathbb{P}(X)$ if X cannot be embedded into the intersection of any two elements of $\mathcal{A}$. We show that if the age of X satisfies the strong amalgamation property, then the structure X can be partitioned into countably many large copies and there is an almost disjoint family of large copies of size continuum. We also show that for a countable ultrahomogeneous poset P, there is a maximal antichain of size continuum in $\mathbb{P}(P)$, while there is a countable maximal antichain in $\mathbb{P}(P)$ if and only if P is not isomorphic to a countable antichain or a disjoint union of infinitely many rational lines. This is joint work with Miloš Kurilić.

Maxwell Levine

Singular Cardinals of Uncountable Cofinality

Much of the recent research in set theory focuses on singular cardinals and their successors because they are the subjects of both independence proofs and direct ZFC theorems. One of the most famous examples is Easton's Theorem: The continuum function $\kappa \mapsto 2^\kappa$ can be any function on the class of regular cardinals as long as it is monotonic and obeys König's Theorem. It was thought that this result could be extended to singular cardinals, but this was refuted by Silver, who proved that GCH cannot fail for the first time at a singular of uncountable cofinality. The behavior of singular cardinals of uncountable cofinality will be the focus of this talk.

We present recent results in broad strokes. The first, which is joint work with Dima Sinapova, concerns the square property at singularized cardinals. It was known that if κ is inaccessible in an inner model V, and if $V \subset W$ where $(\kappa^+)^V = (\kappa^+)^W$ and $(\mathrm{cf}\kappa)^W = \omega$, then $\square_{\kappa,\omega}$ holds in W. However, we find that this does not generalize to the uncountable case: There are models $V \subset W$ in which $V \vDash \kappa$ is inaccessible", $(\kappa^+)^V = (\kappa^+)^W$, and $\omega < (\mathrm{cf}\kappa)^W < \kappa$, and yet $\square_{\kappa,\tau}$ fails in W for all $\tau < \kappa$.

If time allows, we will present a second result, which is joint work with Sy David Friedman, and concerns an Easton-style theorem with regard to the property, "$\kappa \cap \mathrm{cof}(\omega)$ has a non-reflecting stationary subset". The class of cardinals κ that satisfy this property can be essentially any class modulo trivial constraints—for example, it is possible to obtain a model in which this property holds when κ is a successor of a regular cardinal, but fails if κ is inaccessible or if κ is the successor of a singular cardinal. The most challenging case here is that of successors of singulars of uncountable cofinality.

Noah Schoem

Destruction of ideal saturation

An ideal I on κ is κ^+-saturated if every antichain of $(P(\kappa)/I, \leq_I)$ has cardinality $\leq \kappa$, and is κ^+-presaturated if I is precipitous and the forcing $(P(\kappa)/I, \leq_I)$ preserves κ. We answer an open question of [1] of whether there is a forcing extension that destroys κ^+-saturation of ideals on κ while preserving their κ^+-presaturation in the affirmative.

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Jaroslav Šupina

Cardinal invariants and ideal convergence

We discuss several cardinal invariants of the continuum which describe combinatorial properties of ideals on natural numbers. Invariants of our interest appeared during investigations of ideal convergences of sequences of real valued functions or associated covering properties. The talk is based mainly on invariants introduced in [1, 3, 2].

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Dorottya Sziráki

Perfect sets and games on generalized Baire spaces

The generalized Baire space for an uncountable cardinal $\kappa = \kappa^{<\kappa}$ is the space ${}^{\kappa}\kappa$ of functions $\kappa \to \kappa$ equipped with the $<\kappa$-support topology. The study of the topology and descriptive set theory of these spaces is an active area of research, with close connections to many other areas of set theory and to model theory.

The notions of perfectness, scatteredness and the Cantor-Bendixson hierarchy were first generalized to the setting of generalized Baire spaces by Jouko Väänänen, based on certain games of uncountable length. Starting out from concepts introduced by Geoff Galgon, Tapani Hyttinen and Jouko Väänänen, we study some different possible generalizations of these notions. We investigate in detail the connections between these different generalizations and between the games underlying their definitions. For example, we show that Väänänen's generalized Cantor-Bendixson theorem is equivalent to the κ-perfect set property, and is therefore equiconsistent with the existence of an inaccessible cardinal above κ.

Sourav Tarafder

Foundations of mathematics in a model of paraconsistent set theory

Based on the Boolean-valued model construction of classical set theory, we constructed generalised algebra-valued models in [2]. We defined a three-valued algebra $\mathbf{PS}_3$ such that its logic is paraconsistent [1], and the $\mathbf{PS}_3$-valued model $\mathbf{V}^{(\mathbf{PS}_3)}$ validates the negation-free fragment of ZF [2]. In [3], we studied ordinal numbers in $\mathbf{V}^{(\mathbf{PS}_3)}$.

In this talk, we shall discuss properties of the natural numbers in $\mathbf{V}^{(\mathbf{PS}_3)}$. We consider the ordinal ω (as defined in [3]) as the set of natural numbers and prove that this is an inductive set; from this, we conclude that mathematical induction holds in $\mathbf{V}^{(\mathbf{PS}_3)}$ and discuss the arithmetic of natural numbers in this model. Using the standard definition of sizes of sets via bijective functions, we shall define the notion of cardinality in our model and prove some classical theorems such as Cantor's theorem on the size of the power set of a set.

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Marta Vlasáková

Frege’s attitude towards sets

The modern notion of set is considered to be invented by Georg Cantor and consistently established via some standard axiomatics of set theory. Set theory is primarily a mathematical discipline, but the current classical logic is usually held to be closely connected with it. Though logic has historically operated with “extensions of concepts” which are in some sense similar to Cantorian sets, there are important differences between the both concepts. The main difference consists in dealing with sets as “individuals”, i.e. as objects like any others. The Cantorian notion of set was introduced into logic by Gottlob Frege. I would like to elucidate Frege’s attitude towards sets and their role in logic. Frege did not need the notion of set or extension of a concept for grounding logic at all. He considered the notion to be “something derived, whereas in the concept—as I understand the word—we have something primitive” and “the primitive laws of logic may contain nothing derived”. But he needed the notion for his edifice of logicism. After its collapse due to Russell’s uncovering of a paradox, Frege eventually refused Cantorian sets completely. There are no objects like that.

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3 Model Theory

Thomas Scanlon
Maryanthe Malliaris

Ayse Berkman

Sharp actions of groups in the finite Morley rank context

After introducing basics on permutation groups of finite Morley rank, I plan to focus on sharply 2-transitive and generically sharply n-transitive group actions in the finite Morley rank setting.

Let G be a group acting on a set X and fix a positive integer n. If for any two n-tuples $(x_1, \dots, x_n)$ and $(y_1, \dots, y_n)$ consisting of distinct elements of X, there exists a (unique) $g \in G$ such that $gx_i = y_i$ for all $i = 1, \dots, n$, then we say G acts (sharply) n-transitively on X.

For any field (or more generally, for any near-field) K, the action of the group of affine K-linear transformations on K viewed as an affine line, that is $K^* \ltimes K^+ \curvearrowright K$, is sharply 2-transitive. We call such actions standard sharply 2-transitive actions. Sharply 2-transitive finite groups were classified by Zassenhaus in 1936. For a long time, it had been an open question whether every infinite sharply 2-transitive group is standard or not. Finally in 2017, Rips, Segev and Tent, in 2016, Tent and Ziegler; constructed examples of sharply 2-transitive groups which are not standard. However, their examples are not of finite Morley rank. Hence the problem remains open in the finite Morley rank context.

In my talk, first I shall talk about the following partial solution to the problem.

Theorem. (Altınel, B., Wagner, 2019) *Let G be an infinite sharply 2-transitive group of finite Morley rank, and of characteristic p. Then the following holds.*
(a) *If $p = 3$, then G is standard.*
(b) *If $p = 2$, then G splits.*
(c) *If $p \neq 2$ and G splits, then G is standard.*

In a sharply 2-transitive group, if the stabilizer of an element has no involutions, then we say that the characteristic of the group is 2. Otherwise, all strongly real elements (that is, products of two distinct involutions) are conjugate, and their orders are equal to some prime $p \geqslant 3$, or they are of infinite order. In this case, we say the characteristic of the group is p or 0, respectively. If $G = N \rtimes stab(x)$ for some $x \in X$ and normal subgroup $N \trianglelefteq G$, then we say G splits.

The second part of my talk will be devoted to the study of generically sharply n-transitive groups. More precisely, I shall talk about the following theorem.

Theorem. (B., Borovik, 2018) *Let G be a group of finite Morley rank, and V a connected abelian group of Morley rank n with no involutions. Assume that G acts* *definably and generically sharply n-transitively on V, then there is an algebraically closed field F of characteristic not 2, such that $G \curvearrowright V$ is equivalent to* $\mathrm{GL}_n(F) \curvearrowright F^n$.

If G is sharply transitive on a generic subset of X, then we say G acts generically sharply transitively on X. Similarly, if the induced action of G on X^n is generically sharply transitive, then we say G acts generically sharply n-transitively on X.

Philip Dittmann

Models of the common theory of algebraic extensions of the rational numbers

Although the theory of algebraic extensions of $\mathbb{Q}$ has many properties normally seen as undesirable – for instance it is not computably enumerable, and has many completions with bad stability properties –, it still makes sense to investigate its non-standard models. Using the model theory of local fields, as well as some algebraic ingredients interesting in their own right, one can show that every such "non-standard algebraic" field is dense in all its real and p-adic closures. Along the way, we will encounter the classical notion of the Pythagoras number from field theory, as well as a new p-adic version of the same, inspired by axiomatisations of the universal theory of local fields. As a consequence of the denseness, we obtain a result on definability of the valuation ring in henselian fields whose residue field is a number field.

This is joint work with Sylvy Anscombe and Arno Fehm.

Angus Macintyre

Model theory of adeles. Arithmetic equivalence

Let A_K be the ring of adeles of a number field K. Only after Ax had given his analyses of uniformÊ definability and decidability for the completions of K at the its standard absolute values (fifty years ago) could one give informative analyses of the definability and decidability for the individual A_K. This was first done, early on, by Weisspfenning. Much later Derakhshan and I have given a more algebraic treatment purely in the language of rings. Still, many questions remain unanswered, notably that of definability and decidability uniformly in K. This is related to basic issues of unbounded ramification (going back to Herbrand's work in algebraic number theory). Some of these issues will be sketched, but the main emphasis will be on a question posed in other terms by number theorists more than eighty years ago. The question asks to what extent A_K determines K. It has been known for a long time that A_K does not determine K (up to isomorphism) in general, and much fine structure has been discovered (involving Galois theory, zeta functions, class numbers, etc). In the talk I will give a thorough analysis of elementary equivalence for adele rings, and show that it coincides with isomorphism. I also reformulate some work of the number theorists to show that for any K there are at most finitely many L so that A_K and A_L are isomorphic.Ê

This work is joint with J.Derakhshan (Oxford).

Francesco Parente

Model-theoretic properties of ultrafilters and universality of forcing extensions

In this talk, I will discuss some recent results at the interface between model theory and set theory. The first part will be concerned with model-theoretic properties of ultrafilters in the context of Keisler's order. I will use the framework of 'separation of variables', recently developed by Malliaris and Shelah, to provide a new characterization of Keisler's order in terms of saturation of Boolean ultrapowers. Furthermore, I will show that good ultrafilters on complete Boolean algebras are precisely the ones which capture the maximum class in Keisler's order, answering a question posed by Benda in 1974.

In the second part of the talk, I will report on joint work with Matteo Viale in which we apply the above results to the study of models of set theory. In particular, our work aims at understanding the universality properties of forcing extensions. To this end, we analyse Boolean ultrapowers of H_{ω_1} in the presence of large cardinals and give a new interpretation of Woodin's absoluteness results in this context.

Alice Medvedev, Alexander Van Abel*

The Feferman-Vaught Theorem and products of finite fields

We prove that in a product ring of finite fields, the definable subsets are boolean combinations of ∃∀∃-definable sets. This follows from the Feferman-Vaught Theorem on definability in product structures [2], and Kiefe's quantifier reduction result for finite fields [1]. We obtain via our proof that products of integral domains have the maximum amount of definable subsets allowed by the Feferman-Vaught theorem.

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Andrés Aranda*, David Hartman

MB-homogeneous graphs

A relational structure G is homomorphism-homogeneous if every homomorphism f between finite induced substructures is the restriction of an endomorphism F of G to the domain of f (see [1]). A subtype of homomorphism homogeneity is IB-homogeneity, where each isomorphism between finite substructures extends to a bijective endomorphism of the ambient structure.

A variant of Fraïssé-s theorem for IB-homogeneous structures establishes that the limit is unique up to bi-equivalence (M and N are bi-equivalent if every isomorphism with finite domain in M and image in N extends to a bijective homomorphism $M \to N$), but there exist uncountably many countable IB-homogeneous graphs, and even uncountably many pairwise non-isomorphic IB-homogeneous graphs in the same bi-equivalence class [2]. Thus, the best we can hope for is a classification up to the coarser relation of bimorphism equivalence. We present such a classification, answering a question from [2], as a corollary of a result stating that any connected homomorphism-homogeneous graph that does not contain the Rado graph as a spanning subgraph has finite independence number.

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John Baldwin

On strongly minimal Steiner systems: Zilber's conjecture, universal algebra, and combinatorics

With Gianluca Paolini [1], we constructed, using a variant on the Hrushovski dimension function, for every $k \geq 3$, 2^{μ} families of strongly minimal Steiner k-systems. We study the mathematical properties of these counterexamples to Zilber's trichotomy conjecture rather than thinking of them as merely exotic examples. In particular the long study of finite Steiner systems is reflected in results that depend on the block size k. A quasigroup is a structure with a binary operation such that for each equation $xy = z$ the values of two of the variables determines a unique value for the third. The new Steiner 3-systems are bi-interpretable with strongly minimal Steiner quasigroups. For $k > 3$, we show the pure k-Steiner systems have 'essentially unary definable closure' and do not interpret a quasigroup. But we show that for q a prime power the Steiner q-systems can be interpreted into specific sorts of quasigroups, block algebras. This show a dichotomy within the class of strongly minimal sets with flat geometries.

We extend the notion of an (a, b)-cycle graph arising in the study of finite and infinite Stein triple systems ([2]) by introducing what we call the (a, b)-path graph of a block algebra. We exhibit theories of strongly minimal block algebras where all (a, b)-paths are infinite and others in which all are finite only in the prime model. We show how to obtain combinatorial properties (e.g. 2-transitivity) by either varying the basic collection of finite partial Steiner systems or modifying the μ function which ensures strong minimality.

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Emanuele Bottazzi*

An existence result for a class of partial differential equations

We study a nonstandard formulation for time-dependent partial differential equations of the form

$$\begin{cases} u_t = f\,(u, P_1(u), \dots, P_n(u)) & \text{in } \Omega \subseteq \mathbb{R}^k \\ u(0) = u_0 \end{cases} \tag{3.1}$$

with $P_i(u) = \sum_{\alpha \in I_i} a_\alpha \frac{\partial^{|\alpha|} u}{\partial x^\alpha}$ and with distributional or measure-valued initial data u_0. Equations of this form include linear and nonlinear diffusion and systems of conservation laws. Despite their similarities, many of these problems are studied with different techniques and have different notions of solutions [1, 2, 3].

Working in the setting of Robinson's nonstandard analysis, we discretize the differential operators P_i in space by means of finite differences of an infinitesimal step ε: the resulting hyperfinite system of ODEs is formally equivalent to (1). If f is Lipschitz continuous, this system has a unique solution that induces a standard solution to problem (1). For the forward-backward heat equations, this standard solution coincides with the solution obtained with a vanishing viscosity approach; moreover, it is possible to characterize its asymptotic behaviour [1].

We suggest that this nonstandard formulation could be successfully employed in the study of other classes of problems and might lead to novel qualitative information about their solutions.

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David Bradley-Williams

Canonical invariants for t-stratifications

A classical tool in singularity theory is the notion of a stratification of algebraic subsets of $\mathbb{R}^n$ or $\mathbb{C}^n$. In [1], Immanuel Halupczok has developed the notion of *t-stratification* in the context of sets definable in a valued field. We will present joint work with I. Halupczok, in which we investigate invariants of such stratifications that we associate canonically to definable sets, with particular interest in valued fields such as such as $\mathbb{R}((t))$ and $\mathbb{C}((t))$.

This is joint work with Immanuel Halupczok (HHU Düsseldorf).

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Miguel Campercholi

Dominions in filtral quasivarieties

Let $\mathbf{A} \leq \mathbf{B}$ be structures, and $\mathcal{K}$ a class of structures. An element $b \in B$ is *dominated* by $\mathbf{A}$ relative to $\mathcal{K}$ if for all $\mathbf{C} \in \mathcal{K}$ and all homomorphisms $g, g' : \mathbf{B} \to \mathbf{C}$ such that g and g' agree on A, we have $gb = g'b$. Write $\mathcal{D}_{01}$ for the class of bounded distributive lattices, let $\mathbf{B} := \mathbf{2} \times \mathbf{2}$, and let $\mathbf{A}$ be the sublattice of $\mathbf{B}$ with universe $\{\langle 0,0\rangle, \langle 0,1\rangle, \langle 1,1\rangle\}$. As 01-lattice homomorphisms map pairs of complemented elements to pairs of complemented elements, and every element in a distributive lattice has at most one complement, it follows that $\langle 1,0\rangle \in \mathrm{dom}_{\mathbf{B}}^{\mathcal{K}}\mathbf{A}$. The key element to take away from this argument is that $\langle 1,0\rangle$ is generated by A if we add the complementation operation to $\mathbf{B}$. Since this (partial) operation is defined in every member of $\mathcal{D}_{01}$ by the conjunction of atomic formulas

$$\varphi(x,y) := x \wedge y = 0 \,\&\, x \vee y = 1,$$

it is preserved by all relevant maps. This situation generalizes as follows. Recall that a class of algebraic structures is a *quasivariety* provided it is axiomatizable and closed under direct products and substructures. A quasivariety $\mathcal{Q}$ is *filtral* if it is semisimple, its class of simple members is universal, and it is congruence distributive. For instance, $\mathcal{D}_{01}$ is a filtral quasivariety. In our talk we shall discuss the following result and some applications.

Theorem. *Let $\mathcal{Q}$ be a filtral quasivariety and let $\mathcal{M}$ be its class of simple members. Suppose $\mathcal{M}$ has the amalgamation property and $\mathcal{M}_{ec}$ (the class of existentially closed members in $\mathcal{M}$) is axiomatizable. For all $\mathbf{A}, \mathbf{B} \in \mathcal{Q}$ such that $\mathbf{A} \leq \mathbf{B}$ and all $b \in B$ the following are equivalent:*

1. $b \in \mathrm{dom}_{\mathbf{B}}^{\mathcal{Q}}\mathbf{A}$
2. *There are a conjunction of atomic formulas $\delta(\bar{x}, y)$ and $\bar{a} \in A$ such that:*
   - *$\delta(\bar{x}, y)$ defines a function in $\mathcal{Q}$*
   - $\mathbf{B} \vDash \delta(\bar{a}, b)$
   - $\mathcal{M}_{ec} \vDash \forall \bar{x} \exists y\, \delta(\bar{x}, y)$.

Christian Espindola

Preservation theorems for strong first-order logics

We solve an open problem dating back to 1977 mentioned in an article by John P. Burgess, namely "Descriptive set theory and infinitary languages". From the last paragraph:

"One large problem in the model theory of strong first-order languages remains open, which does not lend itself to abstract, descriptive-set-theoretic statement: Can we prove for, say, $\mathcal{L}_{\omega_1,G}$, that any sentence preserved under substructure (resp. homomorphic image) is equivalent to a universal (resp. positive) sentence?"

We answer positively the question providing preservation results for this particular game logic. These are generalizations of the theorems of Łoś-Tarski (resp. Lyndon) on sentences preserved by substructures (resp. homomorphic images). The solution, in ZFC, is then extended to several variants of strong first-order logic that do not satisfy the interpolation theorem; instead, the results on infinitary definability are used. Another consequence of our approach is the equivalence of the Vopěnka principle and a general definability theorem on subsets preserved by homomorphisms.

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Sam van Gool

Monadic second order logic as a model companion

We present a connection between monadic second order logic and first order model theory, which is emerging in our ongoing joint work with Silvio Ghilardi [1, 2].

Monadic second order (MSO) logic, when interpreted in discrete structures, is closely related to certain formal models of computation. For example, the MSO-definable sets of colored finite linear orders (*words*) are exactly the regular languages from automata theory. MSO logic and its connection to automata has been studied on many more structures, including colorings of ω and of trees.

A fundamental insight due to Robinson was that the theory of algebraically closed fields can be generalized to a purely logical notion of *existentially closed model*. The syntactic counterpart of this notion is called the *model companion* of a first order theory. We prove that MSO logic, both on ω-words [1] and on binary trees [2], can be viewed as the model companion of a finitely axiomatized universal first order theory. In each case, this universal theory is closely connected to well-known modal fix-point logics.

Finally, we will point to our ongoing and future work on trees, in which we aim to obtain a similar result for full MSO on trees, whereas our previous results on trees only covered MSO on binary trees and bisimulation-invariant MSO on arbitrary trees.

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Jan Hubička*, Matěj Konečný, Jaroslav Nešetřil

On Hrushovski properties of Hrushovski constructions

Class $\mathcal{K}$ of finite L-structures has the *extension property for partial automorphisms* (*EPPA* or *Hrushovski property*) if for every structure $\mathbf{A} \in \mathcal{K}$ there exists a structure $\mathbf{B} \in \mathcal{K}$ such that $\mathbf{A} \subseteq \mathbf{B}$ and every partial automorphism of $\mathbf{A}$ (that is, an isomorphism of its substructures) extends to automorphism of $\mathbf{B}$.

It was shown by Hrushovski in 1992 that the class of all finite graphs has EPPA. Thanks to numerous applications in group theory and topological dynamics the search for more classes with EPPA continues since then. The Herwig–Lascar theorem, a deep result in the area, provides a structural condition for a class to have EPPA and has been used to prove EPPA for most known examples.

Recently a new link to the structural Ramsey theory has been established. The condition given by the Herwig–Lascar theorem is almost identical to a condition given in [3] for the existence of a precompact Ramsey expansion. In [1], a class $\mathcal{C}_F$ (constructed using the Hrushovski predimension construction) is studied, giving a counterexample to questions in Ramsey theory. It is shown that $\mathcal{C}_F$ has no EPPA, however, it is conjectured that a certain expansion (adding an orientation of edges) has EPPA.

We prove this conjecture using a new strengthening of the Herwig–Lascar theorem [2]. Because $\mathcal{C}_F$ has non-unary algebraic closures, new techniques needs to be developed. In particular, we work with a category of structures in languages equiped with a permutation group on the symbols. We discuss these new techniques and their applications.

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A. R. Yeshkeyev, A. K. Issayeva*

The principle of a "rheostat of atomicity" in the study of AAP models

In this abstract, we want to share with the results concerning the study of countable algebraically prime and atomic models in the sense of studying inductive generally speaking incomplete theories.

Further we will have deal with countable language and some different subclasses of Jonsson theories.

Let AAP be a fixed semantic property, $AAP \in \{\text{atomicity, algebraically primeness}\}$.

Principle of "rheostat". Let two countable models A_1, A_2 of some Jonsson theory T be given. Moreover, A_1 is an atomic model in the sense of [1], and X is $(\nabla_1, \nabla_2) - cl$ -algebraically prime set of theory T and $cl(X) = A_2$.

By the definition of (∇_1, ∇_2) - algebraic primeness of the set X, the model A_2 is in the same time existentially closed and algebraically prime. Thus, the model A_2 is isomorphically embedded in the model A_1. Since by condition the model A_1 is countably atomic, then according to the Vaught's theorem, A_1 is prime, i.e. it is elementarily embedded in the model A_2. Thus, the models A_1, A_2 differ from each other only by the interior of the set X. This follows from the fact that any element of $a \in A_2 \backslash X$ implements some principle type, since $a \in cl(X)$. That is, all countable atomic models in the sense of [1] are isomorphic to each other, then by increasing X we find more elements that do not realize the principle type and, accordingly, $cl(X)$ is not an atomic model in the sense of [1]. Thus, the principle of rheostat is that, by increasing the power of the set X, we move away from the notion of atomicity in the sense of [1] and on the contrary, decreasing the power of the set X we move away from the notion of atomicity in the sense of [2].

In according above mentioned notions we have some numbers of theorems. Those results very close to investigation around atomicity and algebraically primeness in the frame of [2]. Nevertheless even if algebraically primeness is the same, but the combinations of AAP-atomicity differs from atomicity from [2].

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Aleksander Iwanow

Pseudocompact unitary representations of finitely generated groups

We consider unitary representations of finitely generated groups as continuous metric structures ([1]) which are obtained from Hilbert spaces over $\mathbb{C}$ by adding some unitary operators. It is not known if any unitary representation is elementarily equivalent to an ultraproduct of finite dimensional unitary representations (i.e. if its unit ball is pseudocompact). We connect this problem with the topic of approximations by metric groups (in particular with property MF). We also consider appropriate algorithmic problems concerning continuous theories of natural classes of these structures.

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Tomáš Jakl

On connections between logic on words and limits of graphs

Trying to adapt decidability tools from regular languages to more general complexity classes, has naturally led to the study of applications of duality-theoretic methods in the logic on words. In order to recognise a non-regular language one has to construct a syntactic object by the so-called codensity construction [1].

In a completely different discipline, in the limit theory of graphs, it is an ongoing open problem to find a suitable limit object for sequences of finite graphs. This has important applications, for example, when modelling computer networks or biological systems. Taking limits of graphs in the space of finitely additive measures generalises many previous approaches [2].

In this talk I will explain how those two seemingly different theories, in fact, are a special case of one general construction. I will also mention how employing duality-theoretic techniques helps us understand the situation better. By doing so the space of measures is understood simply as a spectrum of the Lindenbaum-Tarski algebra for the First-Order logic extended with new quantifiers.

Note that this theory generalises to arbitrary classes of finite structures.

(This is a joint work with Mai Gehrke and Luca Reggio.)

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Hirotaka Kikyo

On automorphism groups of Hrushovski's pseudoplanes in rational cases

Hrushovski constructed pseudoplanes corresponding to irrational numbers which refute a conjecture by Lachlan [2]. Hrushovski's construction is valid for any real numbers α with $0 < \alpha < 1$. The automorphism groups of the pseudoplanes corresponding to rational numbers α with $0 < \alpha < 1$ are simple groups.

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David M. Evans, Jan Hubička, Matěj Konečný*, Yibei Li

Simplicity of automorphism groups of generalised metric spaces

We study automorphism groups of homogeneous generalised metric spaces $\mathbb{F}$, where the distances come from a partially ordered commutative semigroup $\mathfrak{M} = (M; \oplus, \preceq)$ such that the ternary relation $\mathop{\smile\!\!\!\!\!\!|}$ defined on finite subsets of $\mathbb{F}$ by

$$A \underset{C}{\mathop{\smile\!\!\!\!\!\!|}} B \iff (\forall a \in A)\,(\forall b \in B)\,\big(d(a,b) = \inf\nolimits_{\preceq} \{d(a,c) \oplus d(b,c) : c \in C\}\big)$$

is a stationary independence relation as defined by Tent and Ziegler [1]. We adapt the proof of Tent and Ziegler that the automorphism group of the Urysohn ball is simple [2] and prove the same for $2 \leq |M| < \omega$ and *1-supported* $\mathop{\smile\!\!\!\!\!\!|}$.

Such $\mathfrak{M}$-metric spaces were studied by Hubička, Konečný and Nešetřil [3, 4] in the context of Ramsey theory. They generalise most known binary symmetric homogeneous structures and in particular, as a corollary, we obtain simplicity of the automorphism groups of Cherlin's finite-diameter primitive 3-constrained metrically homogeneous graphs and a strengthening of the results of Li [5].

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Thomas Kucera*, Marcos Mazari-Armida

On universal modules with pure embeddings

This paper arose out of the realization of the second author that some notions of the theory of abstract elementary classes can be used to generalize a result of Shelah [2, 1.2] concerning the existence of universal reduced torsion-free abelian groups with respect to pure embeddings. The contribution of the first author was limited to helping him expand and extend the results to theories of modules.

We show that certain classes of modules have universal models with respect to pure embeddings.

Theorem. *Let R be a ring, T a first-order theory with an infinite model extending the theory of R-modules and $\mathbf{K}^T = (Mod(T), \leq_{pp})$ (where $\leq_{pp}$ stands for pure submodule). Assume $\mathbf{K}^T$ has joint embedding and amalgamation.*

If $\lambda^{|T|} = \lambda$ or $\forall \mu < \lambda(\mu^{|T|} < \lambda)$, then $\mathbf{K}^T$ has a universal model of cardinality λ.

We begin the study of limit models for classes of R-modules with joint embedding and amalgamation. As a by-product of this study, we characterize limit models of countable cofinality in the class of torsion-free abelian groups with pure embeddings, answering Question 4.25 of [1].

Theorem. *If G is a (λ, ω)-limit model in the class of torsion-free abelian groups with pure embeddings, then $G \cong \mathbb{Q}^{(\lambda)} \oplus \prod_p \overline{\mathbb{Z}_{(p)}^{(\lambda)}}^{(\aleph_0)}$.*

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Junguk Lee*, Daniel Max Hoffmann

Elementary theories of PAC structures via Galois groups

Our main interested objects are PAC structures([5, Definition 3.1]), which generalize perfect PAC fields. We show that the first order theories of PAC structures are determined by their Galois groups. Really, in joint work [3] with Dobrowolski, we showed that if given PAC structures have Galois groups isomorphic over a Galois group of a common substructure, then they are elementary equivalent, which generalizes the elementary equivalence theorem for PAC fields(see [4, Theorem 20.3.3]).

In the sequel work [6], we try to generalize criterion to say the theory of a PAC field is $NSOP_n$ if the theory of the complete system of its Galois group is $NSOP_n$ for $n \geq 1$([1, Theorem 3.9] and [7, Corollary 7.2.7, Proposition 7.2.8]). Zoé's amalgamation theorem with respect to complete systems([1, Theorem 3.1]) is crucial in this criterion.

To generalize this amalgamation theorem, we introduce notions of sorted Galois groups and of sorted complete system of sorted Galois groups. Using sorted complete systems, we generalize Zoé's amalgamation theorem to PAC structures. We also generalize the criterion of $NSOP_n$ for PAC fields to PAC structures.

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Michael Lieberman*, Jiří Rosický, Sebastien Vasey

Weak factorization systems and stable independence

We discuss recent joint work with Rosický and Vasey, [1], which reveals surprising connections between model-theoretic independence notions and the behavior of *weak factorization systems*, which play an important role in the analysis of model categories and in homological algebra. In essence, given a reasonable category $\mathcal{K}$ and family of maps $\mathcal{M}$, the category $\mathcal{K}_\mathcal{M}$ obtained by restricting to the morphisms in $\mathcal{M}$ has a stable independence notion just in case $\mathcal{M}$ forms the left half of a *cofibrantly generated* weak factorization system, i.e. one generated by pushouts and transfinite compositions from a set—rather than a class—of basic maps. We sketch the argument, recalling the category-theoretic generalization of stable nonforking independence from [1], as well as the necessary terminology involving weak factorization systems.

As a particular example, we specialize to the case $\mathcal{K} = R$-Mod and $\mathcal{M}$ a class of homomorphisms with kernels in a fixed subcategory: this generalizes the (abstract elementary) classes of modules N considered by Baldwin-Eklof-Trlifaj, [3], and answers a number of questions from their paper. In particular, we prove that this class is tame and stable whenever it is an AEC.

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Johan Lindberg

Constructive semantics and the Joyal-Tierney representation theorem

In this talk I'll describe an ongoing project of further developing the constructive model theory for geometric and first-order logic using complete Heyting Algebra-valued sets. In particular, we study certain locales constructed from the syntax of the theory, some cases of which can be seen as analogues for geometric logic of certain formal topologies first investigated by T. Coquand with collaborators in [1], [2].

Starting from a geometric theory $\mathbb{T}$, the locales X we construct are such that the geometric morphism into the classifying topos **Set**$[\mathbb{T}]$ from sheaves on X is an open surjection, hence these locales can be used for representing **Set**$[\mathbb{T}]$ in the style of Joyal and Tierney [3]. In fact, our analysis of when this geometric morphism is an open surjection allows us to identify and compare several possible locales that can be used to that end, including spatial ones (a la Butz-Moerdijk [4]) when $\mathbb{T}$ has enough models.

This is joint work with Henrik Forssell, Oslo University.

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Rosario Mennuni

Product of invariant types modulo domination-equivalence

In stable theories it is possible to associate to sufficiently big models a certain monoid obtained by quotienting the semigroup of types with tensor product by a relation called “domination-equivalence”. This equivalence relation was generalised to arbitrary theories in [1], where it was studied in the case of the theory of algebraically closed valued fields and it was shown that every global invariant type is domination-equivalent to a product of types concentrating in the residue field or in the the value group. Unfortunately, domination-equivalence is not always a congruence with respect to the product of invariant types, as shown in [2]. The aim of this talk is to present an instance of this incompatibility, along with a first development of the general theory of this interaction.

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A.R. Yeshkeyev, N.M. Mussina*

Hybrids of classes from Jonsson spectrum

Let A be an arbitrary model of countable language. $JSp(A) = \{T/T$ is Jonsson theory in this language and $A \in ModT\}$ and $JSp(A)$ is said to be the Jonsson spectrum of the model.

Definition. We say that the Jonsson theory T_1 is cosemantic to the Jonsson theory T_2 $(T_1 \bowtie T_2)$ if $C_{T_1} = C_{T_2}$, where C_{T_i} are semantic model of T_i, $i = 1, 2$.

The relation of cosemanticness on a set of theories is an equivalence relation. Then $JSp(A)/\bowtie$ is the factor set of the Jonsson spectrum of the model A with respect to $\bowtie$.

Let us define the essence of the operation of the symbol $\boxdot$ for algebraic construction of models, which will be play important role in the definition of hybrids. Let $\boxdot \in \{\cup, \cap, \times, +, \oplus, \prod_F, \prod_U\}$, where $\cup$-union, $\cap$-intersection, $\times$-Cartesian product, $+$-sum and $\oplus$-direct sum, $\prod_F$-filtered product and $\prod_U$-ultraproduct.

Definition. A hybrid of classes $[T]_1, [T]_2$ is the class $[T]_i \in JSp(A)/\bowtie$ if $Th_{\forall\exists}(C_1 \boxdot C_2) \in [T]_i$, we denote such hybrid as $H([T]_1, [T]_2)$.

Fact. *For the theory $H([T]_1, [T]_2)$ in order to be Jonsson enough to be that $(C_1 \boxdot C_2) \in E_{[T]_i}$, where $[T]_i \in JSp(A)/\bowtie$.*

Finally, the main results are the following theorem.

Theorem. *Let $[T]_1, [T]_2$ be perfect convex existentially prime complete for $\forall\exists$-sentences classes from $JSp(A)/\bowtie$. X_i are $\forall\exists$-dcl-sets in the class $[T]_i$, $i \in \{1, 2\}$, i.e. $X_i \subseteq C_i$, where M_i=$dcl(X_i) \in E_{[T]_i}$, $T_i = Th_{\forall\exists}(M_i)$ are also perfect convex existentially prime complete for $\forall\exists$-sentences Jonsson theories. Then, if their hybrid $H([T]_1, [T]_2)$ is a model consistent with $[T]_i$, then $H([T]_1, [T]_2)$ is a perfect class from $JSp(A)/\bowtie$ for $i = 1, 2$.*

Theorem. *Let $[T]_1, [T]_2$ satisfy the conditions of Theorem 1 and $[T]_1, [T]_2$ be ω-categorical. Then their hybrid $H([T]_1, [T]_2)$ is also a ω-categorical class from $JSp(A)/\bowtie$.*

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Inessa Pavlyuk*, Sergey Sudoplatov

On ranks for families of theories of finite abelian groups

We continue to study families of theories of abelian groups [1] characterizing e-minimal subfamilies [2] for finite abelian groups by Szmielew invariants $\alpha_{p,n}$, β_p, γ_p, ε [4, 5], where p are prime numbers, $n \in \omega \setminus \{0\}$, as well as describing possibilities for the rank $RS(\cdot)$ [2].

We denote by $\mathcal{T}_{A,fin}$ the family of all theories of finite abelian groups.

Theorem. *For any infinite family $\mathcal{T} \subseteq \mathcal{T}_{A,fin}$ the following conditions are equivalent:* (1) *$\mathcal{T}$ is e-minimal;* (2) *$dim(\mathcal{T}) = 1$, i.e., $\mathcal{T}$ does not have independent limit values for Szmielew invariants;* (3) *for any upper bound $\alpha_{p,n} \geq m$ or lower bound $\alpha_{p,n} \leq m$, for $m \in \omega$, there are finitely many theories in $\mathcal{T}$ satisfying this bound; having finitely many theories with $\alpha_{p,n} \geq m$, there are infinitely many theories in $\mathcal{T}$ with a fixed value $\alpha_{p,n} < m$.*

Theorem. *Let α be a countable ordinal, $n \in \omega \setminus \{0\}$. Then there is a subfamily $\mathcal{T} \subset \mathcal{T}_{A,fin}$ such that $RS(\mathcal{T}) = \alpha$ and $ds(\mathcal{T}) = n$.*

The families $\mathcal{T}$ for the proof of Theorem 3 have closures $Cl_E(\mathcal{T})$ inside $\mathcal{T}_{A,fin} \cup \mathcal{T}_{A,pf}$, where $\mathcal{T}_{A,pf}$ is the set of theories of pseudofinite abelian groups, and these closures are d-definable.

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Dmitry Emelyanov, Beibut Kulpeshov, Sergey Sudoplatov*

On compositions of structures and compositions of theories

We consider both compositions of structures and compositions of theories and apply these compositions obtaining compositions of algebras of binary formulas [1].

Let $\mathcal{M}$ and $\mathcal{N}$ be structures of relational languages $\Sigma_{\mathcal{M}}$ and $\Sigma_{\mathcal{N}}$, respectively. We define the composition $\mathcal{M}[\mathcal{N}]$ of $\mathcal{M}$ and $\mathcal{N}$ satisfying $\Sigma_{\mathcal{M}[\mathcal{N}]} = \Sigma_{\mathcal{M}} \cup \Sigma_{\mathcal{N}}$, $M[N] = M \times N$ and the following conditions:

1. if $R \in \Sigma_{\mathcal{M}} \setminus \Sigma_{\mathcal{N}}$, $\mu(R) = n$, then $((a_1, b_1), \dots, (a_n, b_n)) \in R_{\mathcal{M}[\mathcal{N}]}$ if and only if $(a_1, \dots, a_n) \in R_{\mathcal{M}}$;
2. if $R \in \Sigma_{\mathcal{N}} \setminus \Sigma_{\mathcal{M}}$, $\mu(R) = n$, then $((a_1, b_1), \dots, (a_n, b_n)) \in R_{\mathcal{M}[\mathcal{N}]}$ if and only if $a_1 = \dots = a_n$ and $(b_1, \dots, b_n) \in R_{\mathcal{N}}$;
3. if $R \in \Sigma_{\mathcal{M}} \cap \Sigma_{\mathcal{N}}$, $\mu(R) = n$, then $((a_1, b_1), \dots, (a_n, b_n)) \in R_{\mathcal{M}[\mathcal{N}]}$ if and only if $(a_1, \dots, a_n) \in R_{\mathcal{M}}$, or $a_1 = \dots = a_n$ and $(b_1, \dots, b_n) \in R_{\mathcal{N}}$.

The theory $T = Th(\mathcal{M}[\mathcal{N}])$ is called the *composition* $T_1[T_2]$ of the theories $T_1 = Th(\mathcal{M})$ and $T_2 = Th(\mathcal{N})$.

Theorem. *If $\mathcal{M}$ and $\mathcal{N}$ have transitive automorphism groups then $\mathcal{M}[\mathcal{N}]$ has a transitive automorphism group, too.*

By this theorem, $T = Th(\mathcal{M}[\mathcal{N}])$ is transitive, and the operation $\mathcal{M}[\mathcal{N}]$ can be considered as a variant of transitive arrangements of structures [2].

The composition $\mathcal{M}[\mathcal{N}]$ is called *E-definable* if $\mathcal{M}[\mathcal{N}]$ has an $\emptyset$-definable equivalence relation E whose E-classes are universes of the copies of $\mathcal{N}$ forming $\mathcal{M}[\mathcal{N}]$. By the definition, each E-definable composition $\mathcal{M}[\mathcal{N}]$ is represented as a E-combination [3] of copies of $\mathcal{N}$ with an extra-structure generated by predicates on $\mathcal{M}$ and linking elements of the copies of $\mathcal{N}$.

Theorem. *If a composition $\mathcal{M}[\mathcal{N}]$ is E-definable then the theory $Th(\mathcal{M}[\mathcal{N}])$ uniquely defines the theories $Th(\mathcal{M})$ and $Th(\mathcal{N})$, and vice versa.*

Theorem. *If a composition $\mathcal{M}[\mathcal{N}]$ is E-definable then the algebra $\mathfrak{P}_T$ of binary isolating formulas for $T = Th(\mathcal{M}[\mathcal{N}])$ is isomorphic to the composition $\mathfrak{P}_{T_1}[\mathfrak{P}_{T_2}]$*

of the algebras $\mathfrak{P}_{T_1}$ *and* $\mathfrak{P}_{T_2}$ *of binary isolating formulas for* $T_1 = Th(\mathcal{M})$ *and* $T_2 = Th(\mathcal{N})$.

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Sergey Davidov, Senik Alvrtsyan, Davit Shahnazaryan*

Invertible binary algebras principally isotopic to a group

A binary groupoid $Q(A)$ is a non-empty set Q together with a binary operation A. Binary groupoid $Q(A)$ is called quasigroup if for all ordered pairs $(a, b) \in Q^2$ exists unique solutions $x, y \in Q$ of the equations $A(a, x) = b$ and $A(y, a) = b$. The solutions of these equations will be denoted by $x = A^{-1}(a, b)$ and $y = {}^{-1}A(b, a)$, respectively. A binary algebra $(Q; \Sigma)$ is called invertible algebra or system of quasigroups if each operation in Σ is a quasigroup operation.

We obtained characterizations of invertible algebras isotopic to a group or an abelian group by the second-order formula.

Definition. We say that a binary algebra $(Q; \Sigma)$ is isotopic to the groupoid $Q(\cdot)$, if each operation in Σ is isotopic to the groupoid $Q(\cdot)$, i.e. for every operation $A \in \Sigma$ there exists permutations $\alpha_A, \beta_A, \gamma_A$ of Q, that:

$$\gamma_A A(x, y) = \alpha_A x \cdot \beta_A y,$$

for every $x, y \in Q$. Isopoty is called principal if $\gamma_A = epsilon$(ϵ - unit permutation) for every $A \in \Sigma$.

Theorem. *The invertible algebra $(Q; \Sigma)$ is a principally isotopic to the abelian group, if and only if the following second-order formula*

$$A({}^{-1}A(B(x, B^{-1}(y, z)), u), v) = B(x, B^{-1}(y, A({}^{-1}A(z, u), v))),$$

is valid in the algebra $(Q; \Sigma \cup \Sigma^{-1} \cup^{-1} \Sigma)$ for all $A, B \in \Sigma$.

Corollary. *[1] The class of quasigroups isotopic to groups is characterized by the following identity:*

$$x(y\backslash((z/u)v)) = ((x(y\backslash z))/u)v.$$

Theorem. *The invertible algebra $(Q; \Sigma)$ is a principally isotopic to a group if and only if the following second-order formula:*

$$A({}^{-1}A(B(x, z), y), A^{-1}(u, B(w, y))) =$$

$$= A({}^{-1}A(B(w, z), y), A^{-1}(u, B(x, y))).$$

is valid in the algebra $(Q; \Sigma \cup \Sigma^{-1} \cup^{-1} \Sigma)$ for all $A, B \in \Sigma$.

Corollary. *The class of quasigroups isotopic to abelian groups is characterized by the following identity:*

$$((xz)/y)(u\backslash(wy)) = ((wz)/y)(u\backslash(xy)).$$

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Sebastien Vasey

Forking and categoricity in non-elementary model theory

The classification theory of elementary classes was started by Michael Morley in the early sixties, when he proved that a countable first-order theory with a single model in some uncountable cardinal has a single model in all uncountable cardinals. The proof of this result, now called Morley's categoricity theorem, led to the development of forking, a notion of independence jointly generalizing linear independence in vector spaces and algebraic independence in fields and now a central pillar of modern model theory.

In recent years, it has become apparent that the theory of forking can also be developed in several non-elementary contexts. Prime among those are the axiomatic frameworks of accessible categories and abstract elementary classes (AECs), encompassing classes of models of any reasonable infinitary logics. A test question to judge progress in this direction is the forty year old eventual categoricity conjecture of Shelah, which says that a version of Morley's categoricity theorem should hold of any AEC. I will survey recent developments, including the connections with category theory and large cardinals, a theory of forking in accessible categories (joint with M. Lieberman and J. Rosický), as well as the resolution of the eventual categoricity conjecture from large cardinals (joint with S. Shelah).

Pedro H. Zambrano

Tameness in classes of generalized metric structures: quantale-spaces, fuzzy sets, and sheaves

Tameness is a very important model-theoretic property of abstract classes of structures, under the assumption of which strong categoricity ([4, 7]) and stability transfer theorems ([1, 8]) tend to hold. We generalize the argument of Lieberman and Rosicky [5]—based on Makkai and Paré's result on the accessibility of powerful images of accessible functors ([3]) under the existence of a proper class of almost strongly compact cardinalities ([2]) —that tameness holds in classes of metric structures, noting that the argument works just as well for structures with underlying Q-spaces, Q a reasonable quantale. Dropping the reflexivity assumption from the definition of metrics, we obtain a similar result for classes with underlying partial metric spaces: through straightforward translations from partial metrics to fuzzy sets and sheaves, we obtain, respectively, fuzzy and sheafy analogues of this result.

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4 Reflection Principles and Modal Logic

Joost Joosten
David Fernández-Duque

Ali Enayat

Some recent news about truth theories

For a fragment B of PA (Peano arithmetic), $\mathsf{CT}^{-}[\mathsf{B}]$ (compositional truth over B) is the theory formulated in the language of arithmetic augmented with a fresh predicate $\mathsf{T}(x)$ to express: "x is the Gödel number of a true arithmetical sentence". The axioms of $\mathsf{CT}^{-}[\mathsf{B}]$ consist of the axioms of B plus finitely many sentences that stipulate that $\mathsf{T}(x)$ is well-behaved on atomic sentences, and obeys Tarski's familiar compositional clauses guiding the behaviour of the truth predicate. We have known, since the pioneering work of Krajewski, Kotlarski, and Lachlan (1981), that $\mathsf{CT}^{-}[\mathsf{PA}]$ is conservative over PA. In this talk we will discuss the following recent developments:

- Recent joint work [1] of Pakhomov and the author on the equivalence of $\mathsf{CT}^{-}[\mathsf{I}_0 + \mathsf{Exp}] + \mathsf{DC}$ with $\mathsf{CT}_0[\mathsf{PA}]$, where DC is the axiom stating "a disjunction of finitely many sentences is true iff one of the disjuncts is true"; and $\mathsf{CT}_0[\mathsf{PA}]$ is the result of adding the induction scheme for Δ_0-formulae that mention the truth predicate to $\mathsf{CT}^{-}[\mathsf{PA}]$. This result refines earlier work by Kotlarski (1986) and Cieślinski (2010), and shows that $\mathsf{CT}^{-}[\mathsf{PA}] + \mathsf{DC}$ is not conservative over PA, since as demonstrated by Wcisło and Łełyk [3], $\mathsf{CT}_0[\mathsf{PA}]$ proves $\mathsf{Con}(\mathsf{PA})$ (and much more).

- Recent joint work [2] of Łełyk, Wcisło, and the author on the *feasible reducibility* of $\mathsf{CT}^{-}[\mathsf{PA}]$, and certain other canonical untyped truth theories to PA. In particular, this shows that $\mathsf{CT}^{-}[\mathsf{PA}]$ does not exhibit superpolynomial speed-up over PA, in sharp contrast to the superexponential speed-up of $\mathsf{CT}^{-}[\mathsf{B}]$ over B for finitely axiomatizable B.

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Emil Jeřábek

Arithmetical and propositional reflection principles

Reflection principles are established as an important tool in the study of first-order theories of arithmetic. In the realm of strong fragments of arithmetic (say, above $I\Delta_0 + EXP$), this means *first-order reflection principles* expressing the soundness of subsystems of arithmetic itself with respect to formulas of bounded complexity. First-order reflection schemata come in various shapes depending on their purpose (uniform reflection principles, local reflection principles, reflection rules), and since they operate inside with the same language as outside, they can be iterated.

This approach is of no use for weak theories of arithmetic such as fragments of bounded arithmetic $I\Delta_0 + \Omega_1$, since these theories cannot even prove the consistency of the base theory Q. However, fragments of bounded arithmetic can be analyzed using reflection principles for *propositional proof systems*, expressing that tautologies of bounded complexity provable in the system are true under Boolean assignments. Using translation of bounded formulas into propositional language, these reflection principles can be themselves expressed by sequences of propositional tautologies.

In this talk, I will review basic properties of reflection principles in both setups, highlighting what makes them similar and what makes them different.

Fedor Pakhomov

A weak set theory that proves its own consistency

We introduce a weak set theory $\mathsf{H}_{<\omega}$. A formalization of arithmetic on finite von Neumann ordinals gives an embedding of arithmetical language into this theory. We show that $\mathsf{H}_{<\omega}$ proves a natural arithmetization of its own Hilbert-style consistency. Unlike the previous examples (due to Willard [2]) of theories proving their own consistency, $\mathsf{H}_{<\omega}$ appears to be sufficiently natural.

The theory $\mathsf{H}_{<\omega}$ is infinitely axiomatizable and proves existence of all individual hereditarily finite sets, but at the same time all its finite subtheories have finite models. Therefore, our example avoids the strong version of Gödel's second incompleteness theorem (due to Pudlák) that asserts that no consistent theory interpreting Robinson's arithmetic Q proves its own consistency [1]. To show that $\mathsf{H}_{<\omega}$ proves its own consistency we establish a conservation result connecting Kalmar elementary arithmetic EA and $\mathsf{H}_{<\omega}$.

The theory $\mathsf{H}_{<\omega}$ is a first-order theory in the signature with equality $=$, membership predicate $\in$, and unary function $\overline{\mathsf{V}}$. Axioms of $\mathsf{H}_{<\omega}$:

1. $x = y \leftrightarrow \forall z(z \in x \leftrightarrow z \in y)$ (Extensionality);
2. $\exists y \forall z(z \in y \leftrightarrow z \in x \wedge \varphi(z))$ (Separation);
3. $y \in \overline{\mathsf{V}}(x) \leftrightarrow (\exists z \in x)(y \subseteq \overline{\mathsf{V}}(z))$ (Defining axiom for $\overline{\mathsf{V}}$);
4. $\exists x\, \mathsf{Nat}_n(x)$, for all $n \in \mathbb{N}$ (all individual natural numbers exist).

Here the formulas $\mathsf{Nat}_n(x)$ expressing the fact that x is the ordinal n are defined in the usual manner: $\mathsf{Nat}_0(x)$ is $\forall y\ y \notin x$ and $\mathsf{Nat}_{n+1}(x)$ is $\forall y\ (\mathsf{Nat}_n(y) \to \forall z(z \in x \leftrightarrow z = y \vee z \in y))$. The intended interpretation of the function $\overline{\mathsf{V}}$ is $\overline{\mathsf{V}}\colon x \longmapsto \mathsf{V}_\alpha$, where α is least ordinal such that $x \subseteq \mathsf{V}_\alpha$.

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Albert Visser

Löb's logic and the Lewis arrow

My talk reports on research in collaboration with Tadeusz Litak.

In the constructive context, the Lewis arrow does not reduce to the modal box. Moreover, a slight generalization of the Lewis arrow, has contraposed interpretability as a special case.

I will discuss versions of Löb's logic with the Lewis arrow. I will address:

- the definition of various systems,
- Kripke semantics,
- explicit fixed points,
- uniform interpolation (which is at present only known for two special systems),
- arithmetical interpretations.

At the end of the talk, I will briefly present some questions for further research.

Mahfuz Rahman Ansari*, A V Ravishankar Sarma

Constraints on selection function: A critique of Lewis-Stalnakers semantics for counterfactuals

Counterfactual conditionals are the special kind of conditional sentences $P\Box \rightarrow Q$, in which the antecedent is always false. Counterfactual conditionals are statements, asserting that something happens under certain conditions, which are presupposed not to be satisfied in reality. The semantics of counterfactuals has been a challenging task for philosophers, since antiquity. The most celebrated and poplar approach in this direction is the Stalnaker (1968)-Lewis' (1973) possible-worlds semantics. According to Lewis-Stalnaker' semantics, a counterfactual $P\Box \rightarrow Q$ holds when in the nearest possible world with respect to the antecedent, the consequent is also true. This approach is based on the comparative similarity of possible worlds. Despite its mathematical elegance, this approach is not free from problems. There is a gap between intuitive notion of similarity of possible worlds and the criteria provided by Lewis. In this paper, we restrict ourselves to the counterfactual conditionals in which the antecedents are treated as action deliberations. We emphasize on additional constraints that are to be imposed on the selection function that picks the nearest possible world. The present study aims to explore the constraints on selection function and tries to reduce the gap between intuitive understanding of counterfactuals and formal analysis of counterfactuals, based on similarity of possible worlds.

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Sergey Drobyshevich*, Sergei Odintsov

Towards a classification of algebraizable FDE-based modal logics

In [3] a first attempt to compare different known FDE-based modal logics was made; this work motivated a classification of FDE-based modal logics developed in [1, 2]. The main ideas behind this classification were (i) that every modal operator $\circ$ over FDE can be analysed in terms of two modal behaviours — one corresponding to asserting $\circ\varphi$ and one corresponding to rejecting $\circ\varphi$ and (ii) that one can study these two behaviours independently from each other in a modular way. Accordingly, four basic *partially-defined* modalities were introduced: $\forall^+$ ($\exists^-$) and $\exists^+$ ($\forall^-$) correspond to asserting and rejecting $\Box\varphi$ ($\Diamond\varphi$) over FDE, respectively. They are partially-defined insofar only one of two behaviours is explicitly defined for them.

In this work we investigate algebraic semantics for these basic modalities. As it turns out, the minimal systems are not algebraizable and we are forced to extend them slightly. This way we obtain four systems closed under rules:

$$\frac{\varphi \dashv\vdash \psi}{\circ\varphi \dashv\vdash \circ\psi}, \qquad \frac{\sim\varphi \dashv\vdash \sim\psi}{\sim\circ\varphi \dashv\vdash \sim\circ\psi}.$$

We show that they are indeed algebraizable and investigate their equivalent algebraic semantics. We also consider relational semantics for these systems, which involve frames with one accessibility relation and one neighbourhood function to model each modality.

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Joseph Boudou, Martínn Diéguez, David Fernández-Duque*

A Complete Intuitionistic Temporal Logic for Topological Dynamics

Linear temporal logic (LTL) is a poly-modal propositional logic which allows for the representation of various tenses including $\circ$ (‘next’) and $\Diamond$ (‘eventually’), and *dynamical (topological) systems* are pairs (X, S) consisting of the action of a continuous function $S\colon X \to X$ on the topological space X. Dynamical systems naturally provide semantics for the language of LTL by using the function S to interpret $\circ$, $\Diamond$ and the topological structure to interpret implication, thus giving rise to an intuitionistic variant of linear temporal logic. Under this interpretation, it is natural to enrich the language of LTL with a universal modality, $\forall$.

In our talk we will show how this language is expressive enough to capture non-trivial phenomena such as Poincaré recurrence and minimality. We will then introduce a ‘minimal’ axiomatization $ITL^0_{\Diamond\forall}$ for intuitionistic temporal logic and discuss a few (in)completeness results:

1. The logic $ITL^0_{\Diamond\forall}$ with tenses $\circ, \Diamond, \forall$ is sound and complete for
   (a) the class of all dynamical systems, and
   (b) the set of all dynamical systems based on the rational numbers, $\mathbb{Q}$.

   In contrast, $ITL^0_{\Diamond\forall}$ is not complete for interpretations based on $\mathbb{R}^n$.
2. The $\forall$-free fragment $ITL^0_{\Diamond}$ is complete for
   (a) the class of all finite dynamic topological systems,
   (b) the class of dynamical systems based over $\mathbb{R}^n$ for any fixed $n \geq 2$, and
   (c) the class of dynamical systems based on the Cantor space.

   However, $ITL^0_{\Diamond}$ is incomplete for the real line.

Finally, we show that $\Box$ (‘henceforth’) is not definable in terms of $\Diamond$ and discuss some problems and possible approaches to including $\Box$ in our logic.

Emanuele Frittaion

The uniform reflection principle in second order arithmetic

I will discuss the full uniform reflection principle in the context of second order arithmetic. I will show how by formalizing a minimum of infinitary proof theory (ω-logic) in a sufficietly strong fragment of second order arithmetic, such as the reverse mathematics base system known as RCA_0 (recursive comprehension axiom), one can give a proof of the following folklore result. Let T_0 be a finitely axiomatizable subsystem of second order arithmetic as strong as RCA_0. Then adding the uniform reflection principle $\mathrm{RFN}(T_0)$ is equivalent to adding full induction. On the other hand, adding the uniform reflection principle $\mathrm{RFN}(T)$, where T is T_0 together with full induction, is equivalent to adding full transfinite induction up to ε_0

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Luciana Garbayo

Dependence logic & medical guidelines disagreement: an informational (in)dependence analysis

Medical guidelines disagreement is a more general problem in medical decision science whereas medical experts hold true distinct guidelines for diagnostic and/or treatment for same patient profiles. Such state of affairs is associated with disagreement in the interpretation of the body of science that supports guidelines for diagnostic and/or treatment decisions by different accredited medical societies. In order to better support medical decision making and augment clinical decision support, a formal semantics of such disagreement has been first suggested with a path to reason with partially contradictory information using natural language processing techniques, while distinguishing formally disagreement from contradictions with propositional calculus and lattice theory (1). This account has been further developed with sheaves, to provide a computational topological treatment for the representation of multiple sources of information and data transformation with mappings (2). In order to enrich this logical notion of disagreement with informational (in)dependence (43), plural states S are considered for multiple forms of interactions between agents, dependent and independent variables and guidelines statements, while a team semantics is explored, within a dependence logic platform.

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Joost Joosten

Hyperarithmetical Turing progressions

Turing progressions arise by iteratedly adding consistency statements over a sound base theory. Schmerl employed Turing progressions over a weak base system in [2] to gauge the (consistency) strength of certain substantially stronger formal systems thus giving rise to ordinal analyses for these systems. Beklemishev showed in [1] how such analyses can be presented and in large part performed within polymodal provability logics. Beklemishev's method employed arithmetic consistency notions only. In this talk we dwell on new techniques that have been developed to take this further to include hyperarithmetical consistency notions.

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Taishi Kurahashi

Derivability conditions and the second incompleteness theorem

Let T be any recursively axiomatized consistent extension of Peano arithmetic. In his famous paper, Gödel showed that the consistency statement $Con_T \equiv \exists x(Fml(x) \land Pr_T(x))$ cannot be proved in T. In the second volume of *Grundlagen der Mathematik*, Hilbert and Bernays proposed a set of conditions for provability predicates which is sufficient for a version of the second incompleteness theorem. That is, if $Pr_T(x)$ is a Σ_1 provability predicate satisfying their conditions, then $Con_T^0 \equiv \forall x(Fml(x) \land Pr_T(x) \rightarrow \neg Pr_T(\neg x))$ cannot be proved in T. Löb [4] found another set of conditions, and proved the so-called Löb's theorem under his conditions. Löb's theorem immediately implies that $Con_T^1 \equiv Pr_T(\ulcorner 0 \neq 0 \urcorner)$ cannot be proved in T. Notice that for provability predicates, Con_T^0 implies Con_T^1, and Con_T^1 implies Con_T.

Related to derivability conditions and the second incompleteness theorem, we proved the following results.

1. There are new sets of derivability conditions which are sufficient for unprovability of Con_T^0.
2. If a Σ_1 provability predicate $Pr_T(x)$ satisfies the following condition B_2^U, then $Pr_T(x)$ satisfies provable Σ_1-completeness.

   $$B_2^U : \text{If } T \vdash \varphi(\vec{x}) \rightarrow \psi(\vec{x}), \text{ then } T \vdash Pr_T(\ulcorner \varphi(\dot{\vec{x}}) \urcorner) \rightarrow Pr_T(\ulcorner \psi(\dot{\vec{x}}) \urcorner)$$

   This is an improvement of Buchholz's observation [1].
3. Hilbert and Bernays's conditions and Löb's conditions are mutually incomparable.
4. Both of Hilbert and Bernays' conditions and the global versions of Löb's conditions are not sufficient for $T \nvdash Con_T$. This shows that both of Hilbert-Bernays' conditions and Löb's conditions do not accomplish Gödel's original statement of the second incompleteness theorem.

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Mateusz Łełyk

Nonequivalent axiomatizations of **PA** and the Tarski Boundary

We study a family of axioms expressing

$$\text{"All axioms of PA are true."} \tag{$*$}$$

where PA denotes Peano Arithmetic. More precisely, each such axiom states that all axioms *from a chosen axiomatization of* PA are true.

We start with a very natural theory of truth $\mathrm{CT}^{-}(\mathrm{PA})$ which is a finite extension of PA in the language of arithmetic augmented with a fresh predicate T to serve as a truth predicate *for the language of arithmetic*. Additional axioms of this theory are straightforward translations of inductive Tarski truth conditions. To study various possible ways of expressing $(*)$, we investigate extensions of $\mathrm{CT}^{-}(\mathrm{PA})$ with axioms of the form

$$\forall x \ \big(\delta(x) \rightarrow T(x)\big). \tag{$**$}$$

In the above (and throughout the whole abstract) $\delta(x)$ is an arithmetical Δ_0 formula which is proof-theoretically equivalent to the standard axiomatization of PA with the induction scheme, i.e. the equivalence

$$\forall x\big(\mathrm{Prov}_{\delta}(x) \equiv \mathrm{Prov}_{\mathrm{PA}}(x)\big).$$

is provable in $I\Sigma_1$. For every such δ, the extension of $\mathrm{CT}^{-}(\mathrm{PA})$ with axiom $(**)$ will be denoted $\mathrm{CT}^{-}[\delta]$.

In particular we are interested in the arithmetical strength of theories $\mathrm{CT}^{-}[\delta]$. The "line" demarcating extensions of $\mathrm{CT}^{-}(\mathrm{PA})$ which are conservative over PA from the nonconservative ones is known in the literature as the *Tarski Boundary*. So far, there seemed to be the least (in terms of deductive strength) natural extension of $\mathrm{CT}^{-}(\mathrm{PA})$ on the nonconservative side of the boundary, whose one axiomatization is given by $\mathrm{CT}^{-}(\mathrm{PA})$ and Δ_0 induction for the extended language (the theory is called CT_0). In contrast to this, we prove the following result:

Theorem. *For every r.e. theory* Th *in the language of arithmetic the following are equivalent:*

1. $\mathrm{CT}_0 \vdash \mathrm{Th}$
2. *there exists* δ *such that* $\mathrm{CT}^{-}[\delta]$ *and* Th *have the same arithmetical consequences.*

Secondly, we use theories $\mathrm{CT}^{-}[\delta]$ to measure the distance between $\mathrm{CT}^{-}(\mathrm{PA})$ and the Tarski Boundary. We prove

Theorem. *There exists a family* $\{\delta_f\}_{f\in\omega^{<\omega}}$ *such that*

1. *for every* $f \in \omega^{<\omega}$, $\mathrm{CT}^{-}[\delta_f]$ *is conservative over* PA*;*
2. *if* $f \subsetneq g$*, then* $\mathrm{CT}^{-}[\delta_g]$ *properly extends* $\mathrm{CT}^{-}[\delta_f]$*;*
3. *if* $f \quad g$ *then* $\mathrm{CT}^{-}[\delta_g] \cup \mathrm{CT}^{-}[\delta_f]$ *is nonconservative over* PA*.*

Tadeusz Litak

Algebras for preservativity

I overview algebraic aspects of our ongoing work with Albert Visser, both published [1] and unpublished, on systems of *constructive strict implication* a.k.a. *Lewis arrow* **J** [2]. The main motivation to study such systems comes from their arithmetical interpretations, particularly in terms of Σ^0_1-*preservativity* [3, 4]. After providing algebraic semantics for the minimal system iA^-, we give examples of some pleasant applications. They include:

- An algebraic connection between the arithmetical notion of *extension stability* with the standard modal notion of a *subframe logic*, using Wolter's notion of a *describable operation* [5].
- Examples of non-derivability proofs for simple consequences of the explicit scheme for de Jongh-Sambin fixpoints impossible in Kripke semantics.
- Wolter-Zakharyaschev-style transfer of results and techniques for classical bimodal logics to their constructive **J**-counterparts via a suitable variant of the Gödel-McKinsey-Tarski translation [6].
- A unifying perspective on generalizations of Kripke, Veltman and neighbourhood semantics.

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A. V. Ravishankar Sarma

Belief revision based on abductive reasoning

Belief revison is concerned with the adjustment of currently held beliefs in the light of new information, particularly when the old belief are contradicting the new information[3]. This paper discusses the role of abductive reasoning- that is, reasoning in which explanatory hypotheses are formed and evaluated, in the change of beliefs. Recent work in artificial intelligence and Philosophical logic recognizes the importance of abductive reasoning within the process of belief revision, discovery, creativity. The cental idea of the paper is that agent seek explanations together with its justification into the agent's current epistemic state before integrating the new information. In the process, an agent given various potential explanations, need to chose the best possible explanation amongst the other competing explanations. We propose an ordering explanations based on the heirarchies of ordering of beliefs called abductive entrenchment ordering of beliefs. This is modification of Pagnucco, Nayak and Foo's model[2], in two different ways. First it proposes abductive entrenchment based on causal explanation and second, it takes care of some of the semantic propertoes such as causal properties, causal explanation, causal relevance, with the belief revision process. The presence or lack of these semantic properties leads to the better understanding of ordering of explanations. We also insights from Kuhn's[1] exhaustive virtues for the theory choice, including accuracy, consistency, scope, simplicity and fruitfulness.

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Igor Sedlár

Fixpoints in generalized Lambek calculus

Modal logics with fixpoint operators have received considerable attention (e.g. dynamic logic with program iteration [3] or epistemic logic with common knowledge [2]). In this talk we discuss a positive modal logic with a binary update modality $\backslash$ and its fixpoint version $\backslash^*$. The update operator is a generalized version of the left division of the (non-associative) Lambek calculus. The difference is that while the Lambek left division has a relational semantics using a ternary relation on a set, the generalized $\backslash$ uses a ternary relation between elements of a set, subsets of that set and members of that set. Our main technical result is a complete axiomatization of a relational semantics for the logic and a decidability result. The technique used to obtain these results is a modification of the techniques used in the case of more standard fixpoint operators such as program iteration or common knowledge. In a sense, this work generalizes the work of Bimbó and Dunn [1] on relational semantics for the logic of Kleene algebras.

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Ilya Shapirovsky

Satisfiability problems on sums of Kripke frames

The complexity of satisfiability problems in modal logic has been systematically investigated since the 1970s; for many logics (e.g, for the standard systems K, T, K4, S4) this problem is known to be PSPACE-complete [1], [3].

In many cases, PSPACE upper bound can be established using the operation of sum of relational structures (Kripke frames) [2]. Given a family $(\mathsf{F}_i \mid i \text{ in } \mathsf{I})$ of frames indexed by elements of another frame I (of the same signature), the *sum of the frames* F_i*'s over* I is obtained from their disjoint union by connecting elements of i-th and j-th distinct components according to the relations in I. Given a class $\mathcal{F}$ of frames-summands and a class $\mathcal{I}$ of frames-indices, $\sum_{\mathcal{I}} \mathcal{F}$ denotes the class of all sums of F_i's in $\mathcal{F}$ over I in $\mathcal{I}$. In this talk we discuss conditions under which the modal satisfiability problem on $\sum_{\mathcal{I}} \mathcal{F}$ is polynomial space Turing reducible to the modal satisfiability problem on $\mathcal{F}$.

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Amirhossein Akbar Tabatabai

The BHK interpretation: looking through Gödel's classical lens

In 1933, Gödel introduced a provability interpretation for intuitionistic propositional logic, **IPC**, to establish a formalization for the BHK interpretation, reading intuitionistic constructions as the usual classical proofs [1]. However, instead of using any concrete notion of a proof, he used the modal system **S4**, as a formalization for the intuitive concept of provability and then translated **IPC** into **S4** in a sound and complete manner. His work then suggested the problem to find the missing concrete provability interpretation of the modal logic **S4** to complete his formalization of the BHK interpretation via classical proofs.

In this talk, we will develop a framework for such provability interpretations. We will first generalize Solovay's seminal provability interpretation of the modal logic **GL** to capture other modal logics such as **K4**, **KD4** and **S4**. The main idea is introducing a hierarchy of arithmetical theories to represent the informal hierarchy of meta-theories of the discourse and then interpreting the nested modalities in the language as the provability predicates of the different layers of this hierarchy. Later, we will combine this provability interpretation with Gödel's translation to propose a classical formalization for the BHK interpretation. The formalization suggests that the BHK interpretation is nothing but a plural name for different provability interpretations for different propositional logics based on different ontological commitments that we believe in. They include intuitionistic logic, minimal logic and Visser-Ruitenburg's basic logic. Finally, as a negative result, we will first show that there is no provability interpretation for any extension of the logic **KD45**, and as expected, there is no BHK interpretation for the classical propositional logic.

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Bartosz Wcisło

Topological models of arithmetic

Our talk concerns the following question: which topological spaces X can be equipped with continuous operations $S : X \to X$, $+ : X^2 \to X$, and $\times : X^2 \to X$ such that $(X, +, \times, S)$ becomes a model of Peano Arithmetic (PA)?

By a result of Malitz, Mycielski, Reinhardt and (independently) Friedman, every theory in a countable signature has a model X which is a Polish space such that all definable relations are Borel (in fact, $F_\sigma \cap G_\delta$). Of course, there are a number of theories in functional signatures which have interesting topological models, i.e., models in which operations are continuous. Prominent examples include topological groups and rings where X can be chosen to be a particularly well-behaved space, for instance a manifold.

Ali Enayat asked whether there exists a Polish space X such that PA has a topological model $(X, +, \times, S)$. In joint work with Ali Enayat and Joel David Hamkins, we have obtained some partial results concerning this question. In particular, we know that any extension of PA has a topological model whose underlying space are rational numbers $\mathbb{Q}$. We have also shown that no finite-dimensional manifold or a compact Hausdorff space can be a model of PA.

If time allows, we will also present some additional results linking topology to the arithmetical structure of the model. In particular, it can be shown that in every cardinality there are topological spaces X which can be endowed with continuous operations making them models of PA such that not every model of PA (in the same cardinality) can be obtained as a topological model with the underlying space X.

All results in the talk are a joint work of Ali Enayat, Joel David Hamkins, and the author.

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Larisa Maksimova, Veta Yun*

On strong recognizability of the intuitionistic logic

The problems of recognizability and strong recognizability, perceptibility and strong perceptibility in extensions of the minimal logic J are studied. These concepts were introduced in [1]–[3].

Let L_0 be a J-logic and L be a finitely axiomatizable logic containing L_0. Say that L is *perceptible over* L_0 if there is an algorithm verifying for any formula A if the inclusion $L_0 + A \geq L$ holds. L is *strongly perceptible* over L_0 if there is an algorithm verifying for any finite set Rul of axioms and rules of inference if the inclusion $L_0 + Rul \geq L$ holds.

A logic L is *recognizable over* L_0 if there is an algorithm verifying for any formula A the equality $L_0 + A = L$. A logic L is *strongly recognizable over* L_0 if there is an algorithm which for every finite system Rul of axiom schemes and rules of inference decides if the logic $L_0 + Rul$ coincides with L.

Although the intuitionistic logic Int is recognizable over J [1] the problem of its strong recognizability over J is not yet solved.

We prove that Int is strong recognizable and strong perceptible over the minimal pre-Heyting logic $Od = \neg\neg(\bot \longrightarrow p)$ and the minimal well-composed logic $JX = (\bot \longrightarrow p) \vee (p \longrightarrow \bot)$.

In addition let us consider the formula $F = (\bot \longrightarrow p \vee q) \longrightarrow (\bot \longrightarrow p) \vee (\bot \longrightarrow q)$. It is unknown whether the logic $J + F$ is recognizable over J. We prove that the formula F is perceptible over JX.

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5 Proof Theory and Proof Complexity

Ulrich Kohlenbach
Samuel Buss

Bahareh Afshari

An infinitary treatment of fixed point modal logic

Fixed point modal logic deals with the concepts of induction and recursion in a most fundamental way. The term refers to any logic built on the foundation of modal logic that features inductively and/or co-inductively defined operators. Examples range from simple temporal logics (e.g. tense logic and linear time logic) to the highly expressive modal μ-calculus and its extensions.

We explore the proof theory of fixed point modal logic with converse modalities, commonly known as 'full μ-calculus'. Building on nested sequent calculi for tense logics [2] and infinitary proof theory of fixed point logics [1], a cut-free sound and complete proof system for full μ-calculus is proposed. As a result of the framework, we obtain a direct proof of the regular model property for the logic (originally proved in [4]): every satisfiable formula has a tree model with finitely many distinct subtrees (up to isomorphism). Many of the results appeal to the basic theory of well-quasi-orders in the spirit of Kozen's proof of the finite model property for μ-calculus [3].

This talk is based on joint work with Gerhard Jäger (University of Bern) and Graham E. Leigh (University of Gothenburg).

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Olaf Beyersdorff

Proof complexity of quantified Boolean formulas

Proof complexity of quantified Boolean formulas (QBF) studies different formal calculi for proving QBFs and compares them with respect to the size of proofs. There exists a number of conceptually quite different QBF resolution calculi, modelling QBF solving approaches, as well as QBF cutting planes, algebraic systems, Frege systems, and sequent calculi. We give an overview of the relative proof complexity landscape of these systems.

From a complexity perspective it is particularly interesting to understand which lower bound techniques are applicable in QBF proof complexity. While some propositional techniques, such as feasible interpolation [3] and game-theoretic approaches [4], can be lifted to QBF, QBF proof complexity also offers completely different approaches that do not have analogues in the propositional domain. These build on strategy extraction, whereby from a refutation of a false QBF a countermodel can be efficiently constructed. Extracting strategies in restricted computational models (such as bounded-depth circuits) and exhibiting false QBFs where countermodels are hard to compute in the same computational model leads to lower bounds for the size of proofs in QBF calculi.

We explain this paradigm for prominent QBFs [2, 1]. For QBF Frege systems this approach even characterises QBF Frege lower bounds by circuit lower bounds [5]. This provides a strong link between circuit complexity and QBF proof complexity, unparalleled in propositional proof complexity.

This line of research also intrinsically connects to QBF solving as different QBF resolution calculi form the basis for different approaches in QBF solving such as QCDCL [7] and QBF expansion [6]. Thus QBF proof complexity provides the main theoretical tool towards an understanding of the relative power and limitations of these powerful algorithms.

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Sara Negri

Syntax for semantics

A general method is presented for converting semantics into well-behaved proof systems. Previous work has shown that the method works in full generality for Kripke semantics. A number of extensions thereof, covering preferential and neighbourhood semantics, will be surveyed to highlight its uniform features.

Pedro Pinto

Proof mining with the bounded functional interpretation

In the context of the proof mining research program [1][2], the standard tool guiding the extraction of new information from noneffective mathematical proofs is Ulrich Kohlenbach's monotone functional interpretation. In 2005, a different interpretation was introduced by Fernando Ferreira and Paulo Oliva, the bounded functional interpretation [3]. We will look at some of the first applications of this functional interpretation to the proof mining of concrete results. In [4], we explained how certain sequential weak compactness arguments can be eliminated from proof mining and used this ideia to obtain a quantitative version of Bauschke's theorem from [5]. Bounds on the metastability (in the sense of Terence Tao) for variants of the proximal point algorithm were obtained in [6][7][8]. This is partly joint work with Bruno Dinis, Fernando Ferreira and Laurenţiu Leuştean.

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Thomas Powell

A new application of proof mining in the fixed point theory of uniformly convex Banach spaces

Proof mining is a branch of mathematical logic which makes use of proof theoretic techniques to extract quantitative information from seemingly nonconstructive proofs. In this talk, I present a new application of proof mining in functional analysis, which focuses on the convergence of the Picard iterates $(T^n x)_{n\in\mathbb{N}}$ for a class of mappings T on uniformly convex Banach spaces whose fixpoint sets have nonempty interior.

Neil Thapen

Induction, search problems and approximate counting

An important open problem in bounded arithmetic is to show that (in the presence of an oracle predicate) theories with more induction are strictly stronger when it comes to proving sentences of some fixed complexity. In classical fragments of Peano arithmetic, the Π_1 consequences of theories can be separated by consistency statements, and the Π_2 consequences by the growth-rate of definable functions. In bounded arithmetic, neither of these seems to be possible.

I will discuss this problem, and describe some recent progress on it. A particular instance of the problem is to find a $\forall\Sigma_1^b$ sentence which is provable in full bounded arithmetic but not in T_2^2 (that is, with induction restricted to Σ_2^b formulas). In [1] we study the theory APC_2, which allows approximate counting of Σ_1^b sets, and appears to have a broadly similar level of strength to T_2^2. We find such a $\forall\Sigma_1^b$ sentence separating APC_2 from full bounded arithmetic, using a probabilistic oracle construction based on a simplified switching lemma.

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Toshiyasu Arai

Some results in proof theory

Let me report on some recent results in proof theory such as the proof-theoretic strengths of the well-ordering principles and of reflecting ordinals.

Paolo Baldi*, Petr Cintula, Carles Noguera

On classical and fuzzy two-layered modal logics for uncertainty: translations and proof-theory

Formal systems for modeling uncertainty are often presented as modal logics with a two-layered syntax, which does not allow for arbitrary nesting of modality. The lower layer is typically used for representing events, and the upper one for reasoning about the measure of uncertainty at hand (probabilities, belief function etc.).

We are interested in two families of such logics: those employing classical logic on both layers, and those employing a suitable fuzzy logic in the upper layer. In [1] we have provided translations between logics of these two families: in particular, we have shown how a proof system for Łukasiewicz logic, based on hypersequents [4] can be used to provide an explicit faithful translation of a classical two-layered logic for probability, introduced in [2] into a corresponding fuzzy one, introduced in [3]. We will present this result and its implications for a systematic investigation of two-layered modal logics from a proof-theoretic perspective, which is still lacking in the literature.

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Libor Behounek

Non-monotonic abstract multiset consequence relations

By working with *sets* of premises, Tarski consequence relations automatically assume certain structural rules. Consequently, most substructural logics can only be represented Tarski-style as *external* consequence relations that preserve designated values. Closer to the spirit of substructural logics, though, are *internal* consequence relations, representing the validity of substructural implication and using *multisets,* rather than sets, of premises to avoid contraction. This was the route taken by Avron [1] and recently elaborated by Cintula et al. [2]. The latter authors generalize Avron's approach from multisets to abstract relations on dually integral Abelian pomonoids, with a Blok–Jónsson monoidal action representing substitution-invariance. Unlike Avron, though, they assume the monotonicity of entailment, thereby ruling out internal consequence relations of weakening-free substructural logics.

In this contribution we explore several non-monotonic generalizations of the abstract consequence relations of [2], with weaker variants of monotonicity motivated by the resource-sensitive interpretation of weakening-free substructural logics. We illustrate general definitions and results on the primary examples of multiset-to-multiset internal consequence relations for particular substructural logics, but also, e.g., on entailment between real-valued sets of formulae related to Pavelka-style logics.

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Marija Boričić

Suppes–style natural deduction system for classical logic

An elegant way to work with probabilized sentences was proposed by P. Suppes (see [3] and [4]). According to his approach we develop a natural deduction system **NKprob**(ε) inspired by Gentzen's natural deduction system **NK** for classical propositional logic. We use a similar approach as in defining general probability natural deduction system **NKprob** (see [1]). Our system will be suitable for manipulating sentences of the form A^n, where A is any propositional formula and n a natural number, with the intended meaning 'the probability of truthfulness of A is greater than or equal to $1 - n\varepsilon$', for some small $\varepsilon > 0$.

For instance, the rules dealing with conjunction looks as follows:

$$\frac{A^m \quad B^n}{(A \wedge B)^{m+n}}(I\wedge) \qquad \frac{A^m \quad (A \wedge B)^n}{B^n}(E\wedge)$$

and modus ponens:

$$\frac{A^m \quad (A \to B)^n}{B^{m+n}}$$

The system **NKprob**(ε) will be a natural counterpart of our sequent calculus **LKprob**(ε) (see [2]).

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Yong Cheng

The limit of incompleteness for Weak Arithmetics

In this work, I examine the limit of incompleteness for Weak Arithmetics w.r.t. interpretation. For a recursively axiomatizable consistent theory T, I define that $G1$ holds for T iff for any recursively axiomatizable consistent theory S, if T is interpretable in S, then S is incomplete. My question is: can we find a weakest theory w.r.t. interpretation such that $G1$ holds for it?

It is often thought that Robinson's theory $\mathbf{R}$ is such a weakest theory. Given theories S and T, let $S \lhd T$ denote that T interprets S but S does not interpret T (S is weaker than T w.r.t. interpretation). A natural question is: can we find a theory S such that $G1$ holds for S and $S \lhd \mathbf{R}$?

I positively answer this question and show that there are many examples of such a theory S via two different methods. Two main theorems are: (1) for each recursively inseparable pair, there is a theory such that $G1$ holds for it and it is weaker than $\mathbf{R}$ w.r.t. interpretation; (2) for any Turing degree $\mathbf{0} < \mathbf{d} < \mathbf{0}'$, there is a theory U such that $G1$ holds for U, $U \lhd \mathbf{R}$ and U has Turing degree $\mathbf{d}$. As two corollaries, I answer a question from Albert Visser and show that there is no weakest theory below $\mathbf{R}$ w.r.t. Turing degrees such that $G1$ holds for it.

José Espírito Santo, Gilda Ferreira*

An embedding of **IPC** into $\mathbf{F_{at}}$ not relying on instantiation overflow

Since 2006 [2], it is known that intuitionistic proposicional calculus **IPC** can be embedded into system $\mathbf{F_{at}}$ – a restriction of Girard's polymorphic system **F** to atomic universal instantiations. Such embedding relies on the Russell-Prawitz's [4] translation of the connectives bottom and disjunction, $\bot := \forall X.X$ and $A \vee B := \forall X.((A \to X) \wedge (B \to X)) \to X$, and on the phenomenon of *instantiation overflow* [3] - the possibility of deriving in $\mathbf{F_{at}}$ the instantiation of these two universal formulas by any (not necessarily atomic) formula. In the present talk we show that there is an alternative (refined) embedding of **IPC** into $\mathbf{F_{at}}$, still based on the Russell-Prawitz's translation of connectives, but based on the admissability of disjunction and absurdity elimination rules, rather than instantiation overflow. Such alternative embedding works as well as the original embedding at the levels of provability and preservation of proof reduction (both embeddings preserve $\beta\eta$-conversions and map commuting conversions to β-equality) but the alternative embedding is more economical than the original one in terms of the size of the $\mathbf{F_{at}}$ proofs and the length of $\mathbf{F_{at}}$ simulations.

Details of this work can be found on [1].

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Marta Gawek*, Agata Tomczyk

Translation of Sequent Calculus into Natural Deduction for Sentential Calculus with Identity

Suszko's Sentential Logic with Identity (SCI) has many unapparent properties, which have not been studied too closely. One of the properties that makes SCI worthwile is that even though it contains an intensional identity connective, the logic itself is extensional. So far, proof methods established for SCI are Hilbert-style system, sequent calculi (SC) (introduced by both [1] and [2]), dual-tableau system by [4] and DFC-algorithms by [5]. In our talk we will present a natural deduction (ND) system for SCI, being the result of employing Negri's strategy [3] of translating SC rules into ND. Rephrasing of logical rules of SC into ND is executed by auxiliarily expressing ND rules using notation appropriate to SC. For a given rule of inference R_1 it is done so by replacing every formula A in ND's derivation by formula $\Gamma \rightarrow A$. Then, the arrow stands for derivability relation, and Γ for a set of open assumptions A depends on. Dischargeable assumptions (A^m, B^n, and so forth) should be expressed as $A^m, \Delta \rightarrow C$. It should be read as: Δ being any (possibly empty) context, and C being the main formula we aim to infer. We will discuss one set of translated rules, which embraces left SCI-rules, as well as admissible rules encompassing transitivity and symmetry. Proofs of soundness and completeness will be discussed as well.

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Iris van der Giessen

Intuitionistic provability logic

The provability logic GL is obtained by adding Gödel-Löb's axiom $\Box(\Box A \to A) \to \Box A$ to a Hilbert calculus for classical modal logic K. Here, $\Box A$ reads as 'A is provable'. Intuitionistic provablity logic is given by K restricted to intuitionistic tautologies together with Gödel-Löb's axiom. In sequent style, this is obtained by adding the modal rule

$$\frac{\Box\Gamma, \Gamma, \Box A \to A}{\Pi, \Box\Gamma \to \Box A}\ \mathrm{GLR}$$

to a sequent calculus for intuitionistic propositional logic.

We study two calculi for intuitionistic provability logic. One is the terminating version of the other. For both systems we prove the admissibility of the cut rule. One proof uses syntactic methods, the other model-theoretic ones.

One calculus that we study is $GLdrie$. This is the intuitionistic propositional calculus $Gdriei$ together with GLR. We obtain cut-admissibility by applying a syntactic method developed by Valentini [1], using a third induction parameter, called *width*.

The other calculus that we consider is $GLvier$, which is obtained by adding GLR to the terminating system $Gvieri$ [2]. Termination of $GLvier$ is based on a loop-preventing proof search adopted from results by Bílková [3]. Cut-admissibility is shown using a semantic strategy as in [4].

Using these results, we establish Craig interpolation for intuitionistic provability logic. One of our aims is to use the terminating calculus $GLvier$ to prove uniform interpolation.

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Anna Glenszczyk

Intuitionistc control logic: an overview

Intuitionistic Control Logic (ICL) was introduced by Ch. Liang and D. Miller. It adds to Intuitionistic Propositional Logic elements of classical reasoning by adding a new logical constant for falsum. Having two different falsum constants enables to define two distinct negations: an ordinary intuitionistic negation and a new negation defined using the additional falsum, which bears some characteristics of classical negation. As a result it is possible within ICL to type programming language control operators while maintaining intuitionistic implication as a genuine connective.

In our talk we would like to discuss basic properties of ICL compared with those of Intuitionistic and Classical Propositional Logics. In particular we will give description of its monadic fragments. We also show that it is possible to embed ICL into second order propositional modal logic using a modification of Gödel-McKinsey-Tarski translation.

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Andrzej Indrzejczak

Admissibility of Cut for Sequent Calculus related to n-labelled Tableaux

The earliest sequent and tableau calculi for many-valued logics were based on the application of n-sided sequents or n-labelled formulae for each n-valued logic. This intuitively natural approach was independently proposed by many logicians in many variants based on two dual interpretations: verificationist and falsificationist. Although in the setting of two-valued logic a choice of interpretation has no effect on the shape of rules, in case of $n > 2$ values we obtain significantly different calculi. Verificationist interpretation was commonly used by proof-theoretically oriented logicians and usually formulated by means of n-sequent calculi (e.g. Rousseau, Takahashi). Also a general cut elimination theorem for this kind of calculi was provided by Baaz, Fermüller and Zach. Falsificationist interpretation was preferred by logicians focusing on proof-search and formulated usually by means of labelled tableux (e.g. Surma, Suchoń, Carnielli). To the best of our knowledge no constructive proof of cut elimination was provided for the latter kind of calculi. We present a structured sequent calculi which may serve as an uniform framework for dealing with both approaches. The main contribution is a strategy for proving cut admissibility for the calculus based on falsificationist interpretation.

Amirhossein Akbar Tabatabai, Raheleh Jalali*

On the logical implications of proof forms

In [1], Iemhoff introduced a syntactic generic form for a certain class of sequent-style rules that she called focused rules. Intuitively speaking, these rules are the rules in which only one side of the sequents is active and the consequence inherits the atomic formulas of the premises. This introduction then led to the implication that the existence of a terminating sequent calculus consisting of these focused rules and the usual **LJ** axioms implies the uniform interpolation property of the super-intuitionistic logic that the calculus captures. In this talk, we will strengthen this implication in two different directions. First, we lower down the base logic from intuitionistic logic to $\mathbf{FL}_e$ to also cover the whole world of sub-structural logics and secondly we will generalize the syntactic form of the rules to a more general form in which both sides of the rule are allowed to be active. The resulting implication then has two major applications. In its positive side, it provides a uniform method to establish uniform interpolation property for logics $\mathbf{FL}_e$, $\mathbf{FL}_{ew}$, $\mathbf{CFL}_e$, $\mathbf{CFL}_{ew}$, **IPC**, **CPC**, their **K** and **KD**-type modal extensions and some basic non-normal modal logics including **E**, **M**, **MC** and **MN**. On its negative side though, the connection implies that no extension of $\mathbf{FL}_e$ enjoys a certain natural type of terminating sequent calculus unless it has the uniform interpolation property. This negative reading of the result then leads to the exclusion of almost all super-intutionistic logics (except seven of them), the logic **K4** and almost all the extensions of the logic **S4** (except six of them) from having such a reasonable calculus.

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Annika Kanckos

Gentzen's tentative views on constructivism

The development of Gerhard Gentzen's (1909–1945) thoughts is seen through a refinement of the argument of consistency for Peano Arithmetic. His constructivist views are paired with a pursual of a modified Hilbert's program. However, the posthumously published earliest consistency proof from 1935 is by modern research considered to contain a gap in the argument. This is a weaker criticism than the original claim that the proof relies on the fan theorem. While Gentzen reworked the proof using transfinite induction, not as a given principle, but as a provable theorem, the gap of the 1935-proof can also be filled with a bar principle from Brouwerian intuitionism taken as a basic recursive principle.

But following the thoughts of Gentzen; how did he prove the theorem of transfinite induction in his published consistency proofs, his lectures, and letters? Based on his constructivist views in the 1936-paper a clear notion of ordinals as a well-ordering of potential infinites is detectable. The basic notion is described as a recursion on these potential infinities. This crucial proof of transfinite induction in the 1936-paper was not modified in the 1938-version. The reason Gentzen had for not reworking this part of the proof, which he himself considered essential, was that he wanted to delay the concretisation and explication of his methods for proving transfinite induction until he had succeeded in proving the consistency of analysis and knew what was needed for such a proof (see [1, p. 248] translating a letter to Bernays, 17 July 1936). Therefore, Gentzen's views on constructivism have to be considered tentative, because his life ended in 1945, before he had reached his expressed aim.

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Anahit Chubaryan, Artur Khamisyan*

On the proof complexity in two universal proof system for all versions of many-valued logics

Two types of universal propositional proof systems were described in [1] such that propositional proof system for every version of MVL can be presented in both of described forms. The first of introduced systems (US) is a Gentzen-like system, the second one (UE) is based on the generalization of the notion of determinative disjunctive normal form, defined by first coauthor for two-valued logic [2]. The last type proof systems are weak ones with a simple strategist of proof search and we have investigated the quantitative properties, related to proof complexity characteristics in them. In particular, for some class of many-valued tautologies simultaneously optimal bounds (asymptotically the same upper and lower bounds) for each of main proof complexity characteristics (size, steps, space and width) were obtained in the second-type systems, considered for some versions of many-valued logic. Now we investigate the relations between the main proof complexty measures in both universal systems. We prove that the system UE $p-simulates$ the system US, but the system US does not $p-simulate$ the system UE and therefore the systems UE and US do not be $p-equivalent$, but nevertheless some classes of k-tautologies have the same proof complexities bounds in both systems, hence we obtain similar results in Gentzen-like system for the same and for other classes of many-valued tautologies as well.

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Satoru Kuroda

On Takeuti-Yasumoto forcing

In late 1996, G.Takeuti and M.Yasumoto [1] published a paper on applications of forcing method for nonstandard models of bounded arithmetic.

In this talk, we will give a reformulation of their forcing construction in terms of two-sort bounded arithmetic. In particular, we will construct Boolean algebras on which generic extensions are models for theories for subclasses of PTIME such as NC^1 or NL. For instance, let $\mathbb{B}$ be the Boolean algebra whose underlying set consists of Boolean formulas over n inputs where n is a fixed nonstandard number. Then a generic subset of $\mathbb{B}$ constitutes a generic extension which is a model of $\mathbf{VNC}^1$.

It turns out that such generic extensions have close connections with separation problems of complexity classes in the ground model. Namely let $\mathfrak{M} \models \mathbf{V}^1$ be a countable nonstandard model which is not closed under exponentiation. Then we can show that $\mathfrak{M} \models (NC^1 = P)$ if and only if any generic extension based on Boolean algebra for NC^1 is a model of $\mathbf{VP}$.

We will also discuss the problem of relating propositional provability in the ground model and the generic extension.

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Martin Maxa

Feasible incompleteness

We will present several conjectures that can be seen as finite counterparts to the well known theorems that are connected to the foundations of mathematics such as Gödel's incompleteness theorems. Their finite versions go already beyond famous open conjectures in computational theory, for example P $\neq$ NP.

José M. Méndez*, Gemma Robles, Francisco Salto

Falsity constants for two independent families of quasi-Boolean logics

In [1], two families of quasi-Boolean logics are defined. One of them is intuitionistic in character; the other one, dual intuitionistic in nature. Both families are determined by a Routley-Meyer ternary relational semantics, negation being interpreted by the "Routley operator" or "Routley star". The aim of this paper is to reconsider the two aforementioned families from the point of view of the same semantics except that now negation will be interpreted by means of two different types of falsity constants following the techniques and strategies discussed in [2]. We will compare the results obtained with those recorded in [1].

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Joachim Mueller-Theys

Multi-valued interpretations

It seems at least unlikely that there is an natural way to extend the $\{0, 1\}$-interpretations of PL to arbitrary values from the unit interval $[0, 1] := \{x \in IR\colon 0 \leq x \leq 1\}$.

For technical reasons, we use the following PL-language: $p_1, p_2, p_3, \dots, \neg\phi, \bigwedge\Phi$, $\bigvee\Psi$, where Φ, Ψ stand for *finite* sets of formulæ previously built. For instance, $\phi \wedge \psi := \bigwedge\{\phi, \psi\}$. $apv(\phi)$ be the set of atomic propositional variables p occurring in ϕ.

Formulæ are binarily interpreted in well-known way, e. g. $V(p) \in \{0, 1\}$, $V'(\neg\phi) = 1$ iff $V'(\phi) = 0$, $V'(\bigwedge\Phi) = 1$ iff $V'(\phi) = 1$ for all $\phi \in \Phi$. $V'(\bigvee\emptyset) = 0$. $\vDash \phi$:iff $V'(\phi) = 1$ for all V, whence $\vDash \phi \leftrightarrow \psi$ iff $V'(\phi) = V'(\psi)$.

Let $apv(\phi) = \{p^1, \dots, p^n\}$. Call $\kappa := \bigwedge\Lambda$ *fundamental* iff either $p^k \in \Lambda$ or $\neg\, p^k \in \Lambda$ for every $1 \leq k \leq n$. $\kappa_1 \perp \kappa_2$ if $\Lambda_1 \neq \Lambda_2$. There is an unique set K of fundamental κ such that $\vDash \phi \leftrightarrow \bigvee K$, where $\delta := \bigvee K$ corresponds to (full) DNF.

Now let $W(p^k) \in [0, 1]$ be any *multi-valued assignment*. The *MVI* or *Buchholz valuation* W' is constructed as follows:

- $W'(p^k) := W(p^k)$,
- $W'(\neg p^k) := 1 - W'(p^k)$,
- $W'(\kappa) := \prod_{\lambda \in \Lambda} W'(\lambda)$,
- $W'(\delta) := \sum_{\kappa \in K} W'(\kappa)$,
- $W'(\phi) := W'(\delta)$;

revealing the somehow evident principles used.

Let $W_{.5} := \frac{1}{2}$. $W'_{.5} = \pi_{PL}$ has been *paradigm* for MVI, whereby *PL-probability* $\pi_{PL}(\phi)$ equals the number of rows with value 1 in the (binary) truth table of ϕ divided by 2^n—originating with the *Tractatus*, probably.

Except for $\pi(\bigwedge\Lambda) = \prod_{\lambda \in \Lambda} \pi(\lambda)$, the principles used to define W' are all properties of probability functions π in the sense of probability logic—maybe of many-valued logic at all—, and we showed that the literal-independent π can be identified with our W'.

However, the nearness to stochastics is deceptive: Consider, e. g., coin toss, where $\pi(head \wedge tail) \neq \pi(head) \cdot \pi(tail)$.

After a joint quest, Wilfried Buchholz had solved the problem technically.—Preliminary versions were presented at ASL-APA and UniLog 2018. Thanks to many participants, Walter Carnielli, Luis Estrada-González, Peter Maier-Borst, Schafag Kerimova, Andreas Haltenhoff.

Ranjan Mukhopadhyay

Cut elimination and Restall's defining rules

The Cut Rule as a structural rule used in sequent calculi can be seen – in the context of justification of deduction – as a recognition of the possibility of indirect proofs for a sentence having logical constant(s). The demand for Cut Elimination Theorem for a calculus having logical constants can be seen, from this perspective, as the demand for showing that if there is an indirect proof for such a sentence then there is a direct proof for it as well. It can be shown that a calculus which has Cut Elimination Theorem for it satisfies Belnap's ("Tonk, Plonk and Plink", 1962) condition of being a conservative extension of the source calculus (S: deducibility as such) comprising of only structural rules including Cut, and the Axiom of Identity. Belnap held that an extended calculus having logical constants should also satisfy the condition of uniqueness.

Restall ("General Existence 1 : Quantification and Free Logic", 2019) takes sequents as proof-theoretic representations of 'clash' between assertions and denials of formulae. Restall shows that his Defining Rules for the classical first order logical constants make way for, not only a conservative extension of S into Classical First Order Predicate (Free) Logic, but also for an uniquely defining extension as well. Restall shows how the usual left/right sequent rules for the constants can be restored from the Defining Rules. For such a restoration Axiom of Identity and the Cut rule become necessary for him. This paper observes that this necessary use of Cut here importantly shows that what is achieved by a Cut Elimination Theorem for a usual calculus (as discussed above) is achieved by Restall's calculus with Defining Rules too, but of course without demanding that Cut be eliminable.

Some ramifications of this feature of Restall's calculus are explored.

Satoru Niki*, Peter Schuster

On Scott's semantics for many-valued logic

Scott [2] proposed abstract entailment relations for a semantics in ordered abelian groups of Łukasiewicz's many-valued logic. Urquhart [3] had a similar semantics. By Scott's entailments one can also represent ideal objects in abstract mathematics [1].

We now show that Scott's semantics fails to be sound for the bottom-to-top direction of Scott's rule $\rightarrow_2$, which was left out from Scott's proof [2, Theorem 3.1]. Indeed,

$$(A \rightarrow B) \rightarrow B \vdash A, B$$

is derivable by Scott's rules but invalid under some interpretation indexed by $[0, \infty)$. Urquhart [4, p. 35] used the same example to show that soundness would fail for his own semantics if one did not require that every formula have a least point of validity. No such request is made by Scott, as it would affect completeness of his semantics.

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Mattias Granberg Olsson*, Graham Leigh

Partial conservativity of $\widehat{\mathrm{ID}}_1^i$ over Heyting arithmetic via realizability.

The result that intuitionistic $\widehat{\mathrm{ID}}_1$ ($\widehat{\mathrm{ID}}_1^{\mathrm{i}}$) is conservative over Heyting arithmetic seems to have been proved only quite recently in a series of papers by Buchholz, Arai, and Rüede and Strahm [3, 1, 4, 2]. We present work in progress on a proposal for a hopefully novel proof of this result, or a substantial part of it, based on realizability and ideas from formal truth. The idea is to use Gödel's diagonal lemma to show that every axiom of some suitable subtheory of $\widehat{\mathrm{ID}}_1^{\mathrm{i}}$ (e.g. of fix-points only for strongly positive operators) is realizable, that realizability respects intuitionistic derivability and that realizability is disquotational for certain classes of formulae (e.g. almost negative formulae).

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Luiz Carlos Pereira*, Edward Hermann Haeusler

New ecumenical systems

Much has been said about the connections between intuitionistic logic and classical logic. Recently, Prawitz (see [4]) proposed a natural deduction *ecumenical* system that puts together classical and intuitionistic logic in a single system, a codification where classical logic and intuitionistic logic can coexist “in peace”. The main idea behind this codification is that classical and intuitionist logic share the constants for conjunction, negation, the absurd, and the universal quantifier, but each has its own disjunction, implication and existential quantifier. Similar ideas are present in Dowek [2] and Krauss [3]. The aims of the present paper are: (i) to present an ecumenical sequent calculus for classical and intuitionistic logic and to state some proof theoretical properties of the system, and (ii) to propose a new ecumenical system, based on the multiple conclusion intuitionistic sequent calculus FIL ([1]), that combines classical logic and the logic of constant domains,

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Franco Parlamento*, Flavio Previale, Federico Munini

The subterm property for some equality sequent calculi

As for *cut elimination* (see [3] p. 93), we say that the *subterm property* holds for a sequent calculus S if there is a "non-trivial" algorithm for transforming a derivation in S of a sequent S into a derivation of S in the same system, that contains only terms occurring in S. We show that the subterm property holds for the following purely equality calculi based on the structural rules:

1. $*\mathbf{EQ}_N$ (N for "natural"), which has the reflexivity axioms $\Rightarrow t = t$ and the multiple congruence rule

$$\frac{\Gamma \Rightarrow r_1 = s_1 \qquad \dots \qquad \Gamma \Rightarrow r_n = s_n \qquad \Gamma \Rightarrow F[v_1/r_1, \dots, v_n/r_n]}{\Gamma \Rightarrow F[v_1/s_1, \dots, v_n/s_n]}$$

2. $*\mathbf{EQ}_B$ (B for "Birkhoff"), which has the reflexivity axioms and the rules:

$$\frac{\Gamma \Rightarrow r = s}{\Gamma \Rightarrow s = r} \qquad\qquad \frac{\Gamma \Rightarrow r = s \qquad \Gamma \Rightarrow s = t}{\Gamma \Rightarrow r = t}$$

$$\frac{\Gamma \Rightarrow r_1 = s_1 \qquad \dots \qquad \Gamma \Rightarrow r_n = s_n}{\Gamma, P[v_1/r_1, \dots, v_n/r_n] \Rightarrow P[v_1/s_1, \dots, v_n/s_n]}$$

$$\frac{\Gamma \Rightarrow r_1 = s_1 \qquad \dots \qquad \Gamma \Rightarrow r_n = s_n}{\Gamma \Rightarrow t[v_1/r_1, \dots, v_n/r_n] = t[v_1/s_1, \dots, v_n/s_n]}$$

3. $*\mathbf{EQ}$, which has the reflexivity axioms and the rules

$$\frac{\Gamma \Rightarrow F[v_1/r_1, \dots, v_n/r_n]}{r_1 = s_1, \dots, r_n = s_n, \Gamma \Rightarrow F[v_1/s_1, \dots, v_n/s_n]}$$

$$\frac{\Gamma \Rightarrow F[v_1/r_1, \dots, v_n/r_n]}{s_1 = r_1, \dots, s_n = r_n, \Gamma \Rightarrow F[v_1/s_1, \dots, v_n/s_n]}$$

where Γ is a finite multiset of formulae, F is a formula, P is an atomic formula different from an equality, r,s, t, the r_i's and s_i's are terms and the v_i's are variable of a first order language and $E[v_i/t_1, \dots, v_i/t_n]$ is used to denote the result of the simultaneous replacement of the free variables $v_1, \dots, v_n$ by the terms $t_1, \dots, t_n$ in the formula or term E.

Moreover, for $*\mathbf{EQ}_N$ and $*\mathbf{EQ}_B$ cut elimination and the subterm property hold simultaneously, namely a derivation in any of such systems of a sequent S can be transformed into a cut-free derivation of S in the same system, containing only terms occurring in S. Although cut elimination holds also for $*\mathbf{EQ}$, it does not hold simultaneously with the subterm property.

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Anahit Chubaryan, Garik Petrosyan*, Sergey Sayadyan

Monotonous and strong monotonous properties of some propositional proof systems for Classical and Non Classical Logics

For some propositional proof system of classical and non-classical logics we investigate the relations between the lines (t-complexities) and sizes (l-complexities) of proofs for minimal tautologies, which are not a substitution of a shorter tautology of this logic, and results of a substitutions in them. For every minimal tautology φ of fixed logic by $S(\varphi)$ is denoted the set of all tautologies, which are results a substitution in φ.

Definition. The proof system Φ is called *t-monotonous* (*l-monotonous*), if for every minimal tautology φ of this system and for every formula ψ from $S(\varphi)$ $t^{\Phi}(\varphi) \leq t^{\Phi}(\psi)$ ($l^{\Phi}(\varphi) \leq t^{\Phi}(\psi)$).

Definition. The proof system Φ is called *t-strong monotonous (l-strong monotonous)*, if for every non-minimal tautology ψ of this system there is such minimal tautology φ of this system such that ψ belong to $S(\varphi)$ and $t^{\Phi}(\psi) \leq t^{\Phi}(\varphi)$ ($l^{\Phi}(\psi) \leq t^{\Phi}(\varphi)$).

Formerly it is proved in [1], that Frege systems for classical and non-classical logics are neither t-monotonous nor l-monotonous.

Now we consider the following systems: propositional resolution systems RC, RI, RJ for classical, intuitionistic and Johansson's logics accordingly, eliminations systems $E?$, EI, EJ, based on the determinative normal forms for the same logics [2], and the system GS, based on generalization of splitting method [3].

Theorem. *The systems RC, RI and RJ are t-strong monotonous (l-strong monotonous), but neither of them is t-monotonous (l-monotonous).*

Theorem. *Each of the systems EC, EI, EJ and GS is neither t-monotonous (l-monotonous) nor t-strong monotonous (l-strong monotonous).*

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Sam Sanders

Umpteen parallel hierarchies and the Gödel hierarchy

We identify natural theorems of higher-order arithmetic that are independent of the medium range of the *Gödel hierarchy* ([7]); this range includes most subsystems of second-order arithmetic. We then obtain a number of independent hierarchies that are parallel to the medium range:

1. The *compactness* hierarchy based on *Cousin's lemma* ([1], 1895).
2. The *Lindelöf* hierarchy based on *Lindelöf's lemma* ([2], 1903).
3. The *local-global* hierarchy based on *Pincherle's theorem* ([6, 5], 1882).
4. The first *net* hierarchy based on the monotone convergence theorem for *nets*, aka Moore-Smith sequences ([3], 1922).
5. The second *net* hierarchy based on moduli of convergence for nets.
6. The *neighbourhood function* hierarchy based on NFP from [4].
7. Variations of these hierarchies.

We work with the Gödel hierarchy based on inclusion and higher-order rather than second-order systems.

This research is part of my joint project with Dag Normann on the Reverse Mathematics and computability theory of the uncountable (see [4] for an introduction).

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Andrei Sipos

Bounds on strong unicity for Chebyshev approximation with bounded coefficients

In the early 1990s, Kohlenbach [1] pursued a program of applying proof-theoretic techniques in order to obtain effective results in best approximation theory, specifically moduli of uniqueness and constants of strong unicity. This was part of a larger research program of Kreisel from the 1950s called 'unwinding of proofs', a program that aimed at applying proof transformations to potentially non-constructive proofs in ordinary mathematics in order to extract new information. The program was later developed by Kohlenbach and his students and collaborators under the name of 'proof mining', extending it to a variety of mathematical areas. For more information on the current state of proof mining, see the book [2] and the recent surveys [3, 4].

What we do is to build up upon the work mentioned above in order to obtain a modulus of uniqueness for best uniform approximation with bounded coefficients, as first considered by Roulier and Taylor [6]. The main novelty is the application of Schur polynomials (for which a reference is [5]) to obtain useful explicit formulas for the interpolation results which are needed in the proof. We present ways these formulas may be bounded and how those bounds may in turn be used to derive and verify the desired modulus.

The results presented in this talk may be found in [7].

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Guillermo Badia, Petr Cintula, Andrew Tedder*

How much propositional logic suffices for Rosser's undecidability theorem?

Rosser [3] famously established that Peano Arithmetic was essentially undecidable against the background of classical first-order logic, that no consistent extension of that theory against that logic was decidable, and this result was extended to Robinson Arithmetic in [4]. We extend the result further by considering not only a weaker arithmetic theory, but a weaker *propositional* logic governing the behaviour of logical connectives. Following Hájek's paper [2] and his later unpublished work on fuzzy arithmetics, we prove essential undecidability for a version of Robinson arithmetic, with predicates (rather than functions) to interpret the arithmetic operations, against the background of a propositional logic with very few logical assumptions. Our logic is much weaker thank the one used by Hájek; it is a variant of Brady's **BBQ** [1] with weakening, in a language including falsum (in terms of which negation is defined), and a crisp (i.e., two-valued) identity predicate. The key logical fact is that, in this system, one can regain enough of the logical inferential machinery to establish the necessary arithmetic facts, the key fact being that all the theory-specific predicates can be shown to be crisp when applied to *numerals*. Therefore, our result suggests that essential undecidability is a property largely dependent on the arithmetic theory, rather than the background propositional logic.

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Andreas Weiermann

A unifying approach to Goodstein principles

The Goodstein principle is arguably the most elementary principle which is independent of first order Peano arithmetic. In our presentation we discuss general properties of Goodstein principles which allow to formulate natural variants of the Goodstein principle which do not depend an a specifc notion of base-k representations of natural numbers. (This is in part joint work with T. Arai, D. Fernández Duque, and S. Wainer.)

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6 Computability

Damir Dzhafarov
Alexander Shen

Laurent Bienvenu*, Barbara Csima, Matthew Harrison-Trainor

Some questions of uniformity in algorithmic randomness

The Ω numbers—the halting probabilities of universal prefix-free machines—are known to be exactly the Martin-Löf random left-c.e. reals [3, 4, 5]. It was previously open however whether this equivalence was uniform, i.e., whether one can uniformly produce, from a Martin-Löf random left-c.e. real α, a universal machine U whose halting probability is α (see for example [1]). We answer this question in the negative. We also answer a question of Barmpalias and Lewis-Pye [2] by showing that given a left-c.e. real α, one cannot uniformly produce a left-c.e. real β such that $\alpha - \beta$ is neither left-c.e. nor right-c.e.

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Wesley Calvert, Douglas Cenzer, Valentina Harizanov*

Approximately computable equivalence structures

In the past, we investigated computable, computably enumerable, and co-computably enumerable equivalence structures and their isomorphisms [2, 3]. In recent years, various authors investigated approximate computability for sets and reducibilities. We introduce and study the notions of generic and coarse computability for equivalence structures and their isomorphisms[1]. A binary relation R on ω is *generically computable* if there is a partial computable function $\varphi : \omega^2 \to \{0,1\}$ such that on its domain, φ coincides with the characteristic function of R and, furthermore, φ is defined on $A \times A$ for a computably enumerable set A of asymptotic density 1. A set $B \subseteq \omega$ is called R-*faithful* if, whenever aRb, then $a \in B$ iff $b \in B$. We say that a generically computable R is faithfully generically computable if the corresponding set A is R-faithful. We show that every equivalence structure has a generically computable copy. We also show that an equivalence structure $\mathcal{E}$ has a faithfully generically computable copy if and only if $\mathcal{E}$ has an infinite faithful substructure with a computable copy.

An equivalence structure $\mathcal{E} = (\omega, E)$ is *coarsely computable* if there is a computable binary relation C such that E and C agree on a set $A \subseteq \omega$ of asymptotic density 1. The structure $\mathcal{E}$ is faithfully coarsely computable if A is both C-faithful and E-faithful. Every equivalence structure has a coarsely computable copy. Not every faithfully coarsely computable equivalence structure has a faithfully generically computable copy, and not every equivalence structure has a faithfully coarsely computable copy. We also investigate generically and coarsely computable isomorphisms and how their categoricity differs from computable categoricity.

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Denis Hirschfeldt

Computability theory, reverse mathematics, and Hindman's Theorem

I will discuss results and open problems concerning the computability-theoretic and reverse-mathematical strength of versions of Hindman's Theorem, which states that for any coloring of the natural numbers with finitely many colors, there is an infinite set S such that all nonempty sums of distinct elements of S have the same color.

Noah Schweber

More effective cardinal characteristics

A *cardinal characteristic of the continuum* is a measure of the difficulty of finding a "sufficiently large" set of reals for a given task - for example, the smallest cardinality of a set of functions from naturals to naturals such that every function is dominated by one in the set, or the smallest cardinality of a non-measurable set. While these are purely set-theoretic objects, they often have computability-theoretic analogues - degree notions which similarly measure the difficulty of creating sufficient sets, but this time from a computational perspective.

In this talk I'll present work, joint with Ivan Ongay-Valverde, on a new class of effective cardinal characteristics. They form the effective analogue of problems such as "How large does a set of 2-branching subtrees of $3^{<\omega}$ have to be in order for every element of 3^{ω} to be a path through one of the trees?" We will show that on the effective side we get multiple distinct hierarchies, and discuss their interactions with classical computability-theoretic notions such as computable traceability.

Time permitting, I'll also say a bit about another less-studied appearance of cardinal characteristics in computability theory - this time, in computable structure theory (this part joint with Uri Andrews, Joe Miller, and Mariya Soskova).

Svetlana Aleksandrova*, Nikolay Bazhenov

On Σ_n^0−classifications

In this talk we will discuss the algorithmic complexity of Σ_n^0−classifications of relations on computable structures.

The notion of classification has developed from the notion of Friedberg enumeration. S. Goncharov and J. Knight [1] considered Friedberg enumerations of classes of structures of some algorithmic complexity. They introduced classification for a class K as a list of structures from K that determines each element of K up to isomorphism, or other equivalence.

This approach can also be used to study classifications of relations on computable structures. Σ_n^0−classification here means a classification of relations defined in the said structure by Σ_n^0−formulae. S. Goncharov and N. Kogabaev in [2] have presented an example of a computable structure without computable Σ_1^0−classification of all unary Σ_1^0−relations. We give a generalisation of this result.

In particular, we show that, while for a given computable structure $\mathfrak{M}$ there is $0^{(n)}$−computable Σ_n^0−classification, for every n one can construct a structure with no $0^{(n-1)}$−computable Σ_n^0−classifications.

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Nikolay Bazhenov*, Manat Mustafa, Mars Yamaleev

Computable reducibility, and isomorphisms of distributive lattices

A standard tool for classifying computability-theoretic complexity of equivalence relations is provided by computable reducibility. Let E and F be equivalence relations on ω. The relation E is *computably reducible* to F, denoted by $E \leq_c F$, if there is a total computable function $f(x)$ such that for all $x, y \in \omega$,

$$(x\, E\, y) \;\Leftrightarrow\; (f(x)\, F\, f(y)).$$

The systematic study of computable reducibility was initiated by Ershov [1, 2].

Let α be a computable non-zero ordinal. An equivalence relation R is Σ^0_α *complete* (for computable reducibility) if $R \in \Sigma^0_\alpha$ and for any Σ^0_α equivalence relation E, we have $E \leq_c R$. The article [3] provides many examples of Σ^0_n complete equivalence relations, which arise in a natural way in recursion theory. In [4], it was proved that for each of the following classes K, the relation of computable isomorphism for computable members of K is Σ^0_3 complete: trees, equivalence structures, and Boolean algebras.

We prove that for any computable successor ordinal α, the relation of Δ^0_α isomorphism for computable distributive lattices is $\Sigma^0_{\alpha+2}$ complete. We obtain similar results for Heyting algebras, undirected graphs, and uniformly discrete metric spaces.

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Michael Stephen Fiske

Quantum Random Self-Modifiable Computation

Among the fundamental questions in computer science, at least two have a deep impact on mathematics. What can computation compute? How many steps does a computation require to solve an instance of the 3-SAT problem? Our work addresses the first question, by introducing a new model called the *ex-machine* [3]. The ex-machine executes Turing machine instructions and two special types of instructions. *Quantum random instructions* are physically realizable with a quantum random number generator [4, 6]. *Meta instructions* can add new states and add new instructions to the ex-machine.

A countable set of ex-machines is constructed, each with a finite number of states and instructions; each ex-machine can compute a Turing incomputable language, whenever the quantum randomness measurements behave like unbiased Bernoulli trials. In 1936, Alan Turing posed the halting problem for Turing machines and proved that this problem is unsolvable for Turing machines. Consider an enumeration $\mathcal{E}_a(i) = (\mathfrak{M}_i, T_i)$ of all Turing machines $\mathfrak{M}_i$ and initial tapes T_i, each containing a finite number of non-blank symbols. Does there exist an ex-machine $\mathfrak{X}$ that has at least one evolutionary path $\mathfrak{X} \rightarrow \mathfrak{X}_1 \rightarrow \mathfrak{X}_2 \rightarrow \ldots \rightarrow \mathfrak{X}_m$, so at the mth stage ex-machine $\mathfrak{X}_m$ can correctly determine for $0 \le i \le m$ whether $\mathfrak{M}_i$'s execution on tape T_i eventually halts? We construct an ex-machine $\mathfrak{Q}(x)$ that has one such evolutionary halting path.

The existence of this path suggests that David Hilbert [5] may not have been misguided to propose that mathematicians search for finite methods to help construct mathematical proofs. Our refinement is that we cannot use a fixed computer program that behaves according to a fixed set of mechanical rules. We must pursue computational methods that exploit randomness and self-modification [1, 2] so that the complexity of the program can increase as it computes.

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Natalia Korneeva

Prefix decidable infinitewords for natural subsets of the set of context-free languages

In the talk we consider prefix decidable infinite words over a finite alphabet for some classes of languages.

Let $\mathcal{L}_{\mathcal{CF}}$ be the class of context-free languages, that is, those that are accepted by finite nondeterministic pushdown automata. Let $\mathcal{L}_{AS}$, $\mathcal{L}_{NS}$ be the classes of languages accepted by finite deterministic pushdown automata by final states or empty stack respectively. Let $\mathcal{L}_{\mathcal{R}}$ be the class of regular languages, that is, those that are accepted by finite automata. It is known that $\mathcal{L}_{\mathcal{R}} \subset \mathcal{L}_{AS} \subset \mathcal{L}_{\mathcal{CF}}$ and $\mathcal{L}_{NS} \subset \mathcal{L}_{\mathcal{CF}}$. Let $\mathcal{L}$ be one of these classes. Also let $Pref(x)$ be the set of prefixes of infinite word x.

Definition. An infinite word x over a finite alphabet Σ is called $\mathcal{L}$-prefix decidable if for any language $L \in \mathcal{L}$ over the alphabet Σ the problem $L \cap Pref(x) \neq \emptyset$ is decidable.

The conception of $\mathcal{L}_{\mathcal{R}}$-prefix decidable infinite words was introduced in [1].

The main results of the talk are relations between classes of prefix decidable infinite words for these classes of languages.

Theorem. *The following conditions for an infinite word x are equivalent:*

1. *x is $\mathcal{L}_{\mathcal{CF}}$-prefix decidable,*
2. *x is $\mathcal{L}_{\mathcal{AS}}$-prefix decidable,*
3. *x is $\mathcal{L}_{\mathcal{NS}}$-prefix decidable.*

Theorem. *There is a $\mathcal{L}_{\mathcal{R}}$-prefix decidable infinite word that is not $\mathcal{L}_{\mathcal{CF}}$-prefix decidable.*

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Robert Lubarsky

Feedback hyperjump

Under feedback computability, the halting problem relative to the halting problem is the halting problem: $X = X'$. Most, if not all, notions of computation that allow for an oracle have a feedback version. The ones that have been explored so far are Turing computability, primitive recursion, and infinite time Turing machines. This talk will include an introduction to feedback, and the current state of knowledge about feedback hyperjump ($X = \mathcal{O}^X$).

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Patrick Lutz*, James Walsh

Descending sequences of hyperdegrees and the second incompleteness theorem

It follows from classical results due to Spector that there is no sequence of reals $A_0, A_1, A_2, \dots$ such that for each n, $A_n \geq_H \mathcal{O}^{A_{n+1}}$. We will give a new proof of this result using the second incompleteness theorem. We will then mention how this fact can be used to give an alternative proof of a result of Simpson and Mummert on a semantic version of the second incompleteness theorem for β_n models. Both of these results seem to suggest a more general connection between well-foundedness of certain partial orders and the second incompleteness theorem. We will mention several other examples of this connection.

Manat Mustafa, Sergey Ospichev*

About Rogers semilattices of finite families in Ershov hierarchy

There is a well-known result, that any finite family of c.e. sets has computable principal numbering[1]. In [2], K.Abeshev shows that there is a finite family of sets in Ershov hierarchy without Σ_2^{-1}-computable principal numbering. With the help of Γ-operator in [3], above result can be generalized to any level(finite and successor ordinals) of Ershov hierarchy. Here we concentrate our interest to different types of Σ_2^{-1}-computable numberings of finite families of Σ_2^{-1}-sets and c.e.-sets. The main result is:

Theorem. *Let $\mathcal{S} = \{A, B\}$ be any family with A, B are c.e. sets with $A \subseteq B$ but $A \setminus B$ is not c.e., then the Rogers semilattice $\mathcal{R}_2^{-1}(\mathcal{S})$ is isomorphic to family L_0^m of all m-degrees of c.e. sets.*

Corollary. *Any Σ_2^{-1}-computable numbering of $\mathcal{S}$ is equivalent to some computable numbering of $\mathcal{S}$.*

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Michal Tomasz Godziszewski, Dino Rossegger, Luca San Mauro*

Quotient presentations of structures

A *c.e. quotient presentation* of a structure $\mathcal{A} = \langle A; \{f_i\}_{i\in I}, \{R_j\}_{j\in J}\rangle$ consists of a structure $\mathcal{A}^* = \langle \mathbb{N}; \{f_i^*\}_{i\in I}, \{R_i^*\}_{j\in J}\rangle$ and a c.e. equivalence relation E (often called a *ceer*) such that the functions of $\mathcal{A}^*$ are uniformly computable, the relations of $\mathcal{A}^*$ are uniformly c.e., E is a congruence with respect to $\mathcal{A}^*$, and $\mathcal{A}^*/E \cong \mathcal{A}$. E *realizes* $\mathcal{A}$ if $(\mathcal{A}^*, E)$ is a c.e. quotient presentation of $\mathcal{A}$, for some $\mathcal{A}^*$; otherwise, E *omits* $\mathcal{A}$. Khoussainov and his collaborators (see, e.g., [2, 3]) investigated, for familiar classes of structures, which structures are realized by a given ceer E. We are interested in the reverse problem, i.e., we study the structure of the following spectra.

Definition. The *spectrum of ceers* of a structure $\mathcal{A}$ is the following class of ceers

$$\mathrm{CeersSp}(\mathcal{A}) = \{E \in \mathbf{Ceers} : E \text{ realizes } \mathcal{A}\}.$$

During the talk, we will discuss the main motivations for the project and we will demonstrate theorems relating the program to the study of some distinguished classes of equivalence relations, such as those considered in [1].

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Rumen Dimitrov, Valentina Harizanov, Andrey Morozov, Paul Shafer*, Alexandra Soskova, Stefan Vatev

Cohesive powers of ω

A *cohesive power* of a computable structure is an effective analog of an ultrapower of the structure in which a cohesive set plays the role of an ultrafilter. We study the cohesive powers of computable copies of the structure $(\omega, <)$, i.e., the natural numbers with their usual order. By a *computable copy of* $(\omega, <)$, we mean a computable linear order $\mathcal{L} = (L, \prec)$ that is isomorphic to $(\omega, <)$, but not necessarily by a computable isomorphism. That is, the successor function of $\mathcal{L}$ may not be computable. Our main findings are the following. First, recall that ζ denotes the order type of the integers, that η denotes the order type of the rationals, and that $\omega + (\eta \times \zeta)$ (often also written $\omega + \zeta\eta$) is familiar as the order type of countable non-standard models of Peano arithmetic.

1. If $\mathcal{L}$ is a computable copy of $(\omega, <)$ with a computable successor function, then every cohesive power of $\mathcal{L}$ has order type $\omega + (\eta \times \zeta)$.

2. There is a computable copy $\mathcal{L}$ of $(\omega, <)$ with a **non**-computable successor function such that every cohesive power of $\mathcal{L}$ has order type $\omega + (\eta \times \zeta)$.

3. Most interestingly, there is a computable copy $\mathcal{L}$ of $(\omega, <)$ (with a necessarily non-computable successor function) having a cohesive power that is **not** of order type $\omega + (\eta \times \zeta)$.

Marta Fiori Carones, Alberto Marcone, Paul Shafer, Giovanni Soldà*

Reverse Mathematics of some principles related to partial orders

In this talk, we will study (some variations of) the following theorem, due to Rival and Sands ([3]) in the context of Reverse Mathematics:

Theorem. *(*RS-po*) Let P be an infinite partial order of finite width k. Then there is an infinite chain C of P such that for every element $p \in P$, p is comparable with* 0 *or infinitely many elements of C.*

In particular, we show that ACA_0, the third of the Big Five subsystems of Z_2, is enough to prove RS-po, although no reversal is known to hold. An interesting result is obtained by fixing the width of the partial order P: if $k = 3$, we prove that the theorem is equivalent to ADS, a combinatorial principle introduced by Hirschfeldt and Shore in [2], and a widely studied element of the "zoo below ACA_0" (a very good presentation of which is given for instance in [1]). Notably, this version of the theorem appears to be the first natural mathematical statement proven to be equivalent to ADS.

Finally, some partial results on a stronger version of RS-po, where we require comparability with 0 or *cofinitely* many elements of C, will be presented.

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Julia Knight, Alexandra Soskova*, Stefan Vatev

Effective coding and decoding structures

Friedman and Stanley introduced Borel embeddings as a way of comparing classification problems for different classes of structures. A Borel embedding of a class K in a class K' represents a uniform procedure for coding structures from K in structures from K'. Many Borel embeddings are actually Turing computable.

When a structure $\mathcal{A}$ is coded in a structure $\mathcal{B}$, effective decoding is represented by a Medvedev reduction of $\mathcal{A}$ to $\mathcal{B}$. Harrison-Trainor, Melnikov, Miller, and Montalbán defined a notion of effective interpretation of $\mathcal{A}$ in $\mathcal{B}$ and proved that this is equivalent with the existing of computable functor, i.e. a pair of Turing operators, one taking copies of $\mathcal{B}$ to copies of $\mathcal{A}$, and the other taking isomorphisms between copies of $\mathcal{B}$ to isomorphisms between the corresponding copies of $\mathcal{A}$. The first operator is a Medvedev reduction. For some Turing computable embeddings Φ, there are *uniform* formulas that effectively interpret the input structure in the output structure.

The class of undirected graphs and the class of linear orderings both lie on top under Turing computable embeddings. The standard Turing computable embeddings of directed graphs (or structures for an arbitrary computable relational language) in undirected graphs come with uniform effective interpretations. We give examples of graphs that are not Medvedev reducible to any linear ordering, or to the jump of any linear ordering. Any graph can be interpreted in some linear ordering using computable Σ_3 formulas. Friedman and Stanley gave a Turing computable embedding L of directed graphs in linear orderings. We show that there do not exist $L_{\omega_1\omega}$-formulas that uniformly interpret the input graph G in the output linear ordering $L(G)$.

Alexey Ryzhkov, Alexey Stukachev*, Marina Stukacheva

Interval semantics for natural languages and effective interpretability over the reals

Among the methods of formal semantics for natural languages [2], interval semantics is used for the formalization of the concepts of tense and aspect of verb forms in sentences, as well as for expressing differences between proper and improper speech [1]. The time scale corresponds to the axis of the real numbers. The basic verbal constructions of natural languages (Russian, English, German, etc.) in interval semantics are expressed by first-order logic formulas over the field of reals.

We apply the methods of effective interpretability over the reals together with the methods of interval semantics for verb constructions for analyzing sentences in Russian.

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Andrey Morozov, Jamalbek Tussupov*

On minimal elements in the Δ-reducibility on families of predicates

Fix some countable set U. By *predicate* here we mean an arbitrary subset of an arbitrary finite Cartesian power of U. We study two kinds of reducibilities on finite families of predicates.

We say that a predicate R is Δ–*definable over the predicates* $P_1, \dots, P_k$ if R itself and its complement can be defined in the structure $\langle U; P_1, \dots, P_k \rangle$ by means of $\exists$–formulas with parameters.

Let $S_0 = \{P_0, \dots, P_{k-1}\}$ and S_1 be two finite families of predicates. We say that S_0 *is* Δ–*definable in* S_1, if all the predicates in S_0 are Δ–definable in S_1 and we denote this fact as $S_0 \leqslant^0_\Delta S_1$. If $S_0 \leqslant^0_\Delta S_1$ and $S_1 \leqslant^0_\Delta S_0$ then we denote this fact as $S_0 \equiv^0_\Delta S_1$. The relation $\leqslant^0_\Delta$ is a preordering, $\equiv^0_\Delta$ is an equivalence and the quotient $\leqslant^0_\Delta / _{\equiv^0_\Delta}$ defines an upper semilattice in which the least upper bound of elements $S_0 / _{\equiv^0_\Delta}$ and $S_1 / _{\equiv^0_\Delta}$ equals to $(S_0 \cup S_1) / _{\equiv^0_\Delta}$ and $\bot^0_\Delta = \emptyset / _{\equiv^0_\Delta}$ is the smallest element. Denote this semilattice by D^0_Δ.

If we consider families of predicates up to isomorphism, we arrive at the notion of Δ–*reducibility on families of predicates*. We say that a finite family of predicates S_0 Δ–*reduces to a finite family* S_1 (and denote this as $S_0 \leqslant_\Delta S_1$), if there exists a finite family of predicates S' such that $S'_0 \leqslant^0_\Delta S_1$ and S'_0 is a conjugate of S_0 by means of some permutation on U.

If $S_0 \leqslant_\Delta S_1$ and $S_1 \leqslant_\Delta S_0$ then we denote this fact as $S_0 \equiv_\Delta S_1$. The quotient $\leqslant_\Delta / _{\equiv_\Delta}$ defines a structure D_Δ, which is a partial order with smallest element $\bot_\Delta = \emptyset / _{\equiv_\Delta}$.

Theorem. 1. *The structure D_Δ fails to be an upper semilattice.*

2. *The families consisting of unary predicates define in D_Δ an ideal of order type ω.*

Theorem. *Each of the structures $D^0_\Delta \setminus \{\bot^0_\Delta\}$ and $D_\Delta \setminus \{\bot_\Delta\}$ contains 2^ω minimal elements.*

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Nikolay Bazhenov, Hristo Ganchev, Stefan Vatev*

Computable embeddings for pairs of linear orderings

Friedman and Stanley [3] introduced the notion of *Borel embedding* to compare complexity of the classification problems for classes of countable structures. Calvert, Cummins, Knight, and Miller [1] (see also [2] and [4]) developed two notions, *computable embeddings* and *Turing computable embeddings*, as effective counterparts of Borel embeddings.

We follow the approach of [1] and study computable embeddings for pairs of structures, i.e. for classes $\mathcal{K}$ containing precisely two non-isomorphic structures. Our motivation for investigating pairs of structures is two-fold. These pairs play an important role in computable structure theory and also they constitute the simplest case, which is significantly different from the case of one-element classes. It is not hard to show that for any computable structures $\mathcal{A}$ and $\mathcal{B}$, the one-element classes $\{\mathcal{A}\}$ and $\{\mathcal{B}\}$ are equivalent with respect to computable embeddings. On the other hand, computable embeddings induce a non-trivial degree structure for two-element classes consisting of computable structures.

In this talk we will concentrate on the pair of linear orders ω and $\omega^\star$. We will use $\deg_{tc}(\{\omega,\omega^\star\})$ to denote the degree of the class $\{\omega,\omega^\star\}$ under Turing computable embeddings. Quite unexpectedly, it turns out that a seemingly simple problem of studying computable embeddings for classes from $\deg_{tc}(\{\omega,\omega^\star\})$ requires developing new techniques.

We give a necessary and sufficient condition for a pair of structures $\{\mathcal{A},\mathcal{B}\}$ to belong to $\deg_{tc}(\{\omega,\omega^\star\})$. We also show that the pair $\{1+\eta,\eta+1\}$ is the greatest element inside $\deg_{tc}(\{\omega,\omega^\star\})$, with respect to computable embeddings. More interestingly, we prove that inside $\deg_{tc}(\{\omega,\omega^\star\})$, there is an infinite chain of degrees induced by computable embeddings.

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Ilya Vlasov

On enumerations of families of sets of computableelements of metric spaces

In this talk, we introduce a definition of enumerations of families of sets of computable elements of a computable Polish space. We have obtained various results which concerns such enumrations. Namely, the Rogers semilattices of the enumerations were described in terms of certain ideals of the Rogers semilattices of enumerations of families of Σ^0_2-sets. A criterion of existence of a universal enumeration is described.

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Nikolay Bazhenov, Dino Rossegger, Luca San Mauro, Maxim Zubkov*

On bi-embeddable categoricity of linear orders

Given a linear order $\mathcal{L}$ and a linear order $\mathcal{M}$ bi-embeddable with $\mathcal{L}$, we say that $\mathcal{M}$ is a bi-embeddable copy of $\mathcal{L}$. We study the complexity of embeddings using the following definition analogous to computable categoricity.

Definition. A countable linear order $\mathcal{L}$ is (relatively) Δ^0_n-bi-embeddably categorical if for any bi-embeddable computable (for any bi-embeddable) copy $\mathcal{M}$, $\mathcal{M}$ and $\mathcal{L}$ are bi-embeddable by Δ^0_n-embeddings ($\Delta_n^{\mathcal{L}\oplus\mathcal{M}}$-embeddings, correspondingly).

Recall, that a linear order is scattered if it has no a suborder of type η. It is easy to see, that the question about the level of bi-embeddable categoricity is nontrivial only for scattered linear orders. We obtain characterization of linear orders with finite levels of bi-embeddable categoricity.

Theorem. *A scattered computable linear order of rank n is relatively Δ^0_{2n}-bi-embeddably categorical, and is not Δ^0_{2n-1}-bi-embeddably categorical.*

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Damir Zainetdinov

Limitwise monotonic reducibility of sets and Σ-definability of abelian groups

In my talk I will consider limitwise monotonic reducibility (lm-reducibility for short) of sets via Σ-definability of abelian groups. The notion of lm-reducibility of sets via the limitwise monotonic operator was introduced in [1]. The main results obtained with the investigation of limitwise monotonic functions, sets, and sequences can be found in papers [2, 3].

Definition. Let sets $A, B \subseteq \mathbb{N}$. We define the following family of initial segments:

$$\mathcal{F}(A) = \{\mathbb{N} \upharpoonright n : n \in A\}.$$

Then $A \leq_{lm} B \Leftrightarrow \mathcal{F}(A) \sqsubseteq_{\Sigma} \mathcal{F}(B)$, where definition of Σ-reducibility on the families can be found in [4].

We consider an abelian group $G(A)$ in the following form:

$$G(A) = \bigoplus_{n \in A} \left(\bigoplus_{m \in \mathbb{N}} \mathbb{Z}_{p^n} \right),$$

where $\mathbb{Z}_{p^n}$ – cyclic group of order p^n and p is prime.

The main result of my talk is to obtain a description of the lm-reducibility of sets on the language of Σ-definability of abelian groups.

Theorem. *The family $\mathcal{F}(A)$ is Σ-definable in the hereditarily finite superstructure $\mathbb{HF}(G(A))$ over the group $G(A)$.*

Theorem. *Let $A, B \subseteq \mathbb{N}$. Let $G(A)$ and $G(B)$ be abelian groups defined for sets A and B, respectively. Then $A \leq_{lm} B$, if and only if the group $G(A)$ is Σ-definable in the hereditarily finite superstructure $\mathbb{HF}(G(B))$.*

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7 Foundations of Geometry

John T. Baldwin
Victor Pambuccian

Michael Beeson

On the notion of equal figures in Euclid

Euclid uses an undefined notion of "equal figures", to which he applies the common notions about equals added to equals or subtracted from equals. When we formalized Euclid Book I for computer proof-checking, we had to add fifteen axioms about undefined relations "equal triangles" and "equal quadrilaterals" to replace Euclid's use of the common notions. In this paper, we offer definitions of "equal triangles" and "equal quadrilaterals, that Euclid could have given, and prove that they have the required properties, by proofs Euclid could have given. This removes the need for adding new axioms.

Pierre Boutry

Towards an independent version of Tarski's system of geometry

In 1926-1927, Tarski designed a set of axioms for Euclidean geometry which reached its final form in a manuscript by Schwabhäuser, Szmielew and Tarski in 1983. The differences amount to simplifications obtained by Tarski and Gupta. Gupta presented an independent version of Tarski's system of geometry, thus establishing that his version could not be further simplified without modifying the axioms. To obtain the independence of one of his axioms, namely Pasch's axiom, he proved the independence of one of its consequence: the previously eliminated symmetry of betweenness. However, an independence model for the non-degenerate part of Pasch's axiom was provided by Szczerba for another version of Tarski's system of geometry in which the symmetry of betweenness holds. This independence proof cannot be directly used for Gupta's version as the statements of the parallel postulate differ.

In this talk, we present our progress towards obtaining an independent version of a variant of Gupta's system. Compared to Gupta's version, we split Pasch's axiom into this previously eliminated axiom and its non-degenerate part and change the statement of the parallel postulate. To select this statement, our previous paper, *Parallel postulates and continuity axioms: a mechanized study in intuitionistic logicusing Coq*, proved to be useful so we detail some of these results.

John Mumma

Diagrams and parallelism

The topic of my talk is how the relation of parallelism is represented in diagrammatic proofs of plane elementary geometry. I will discuss how the boundedness of diagrams motivates a constructive conception of the relation, and consider how the formal system presented in [1] can be modified in accord with this conception.

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Gianluca Paolini

First-order model theory of free projective planes

We prove that the theory of open projective planes is complete and strictly stable, and infer from this that Marshall Hall's free projective planes $(\pi^n : 4 \leq n \leq \omega)$ are all elementary equivalent and that their common theory is strictly stable and decidable, being in fact the theory of open projective planes. We further characterize the elementary substructure relation in the class of open projective planes, and show that $(\pi^n : 4 \leq n \leq \omega)$ is an elementary chain. We then prove that for every infinite cardinality κ there are 2^κ non-isomorphic open projective planes of power κ, improving known results on the number of open projective planes. Finally, we characterize the forking independence relation in models of the theory and prove that π^ω is strongly type-homogeneous.

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Piotr Blaszczyk

Axioms for Euclid's Elements book V, their consequences and some independence results

Euclid's *Elements*, book V develops the theory of proportions as applied to magnitudes; it is the key theory for understanding Greek and early modern mathematics. By formalizing its definitions and the tacit assumptions behind its proofs, we reconstruct book V with its 25 propositions as an axiomatic theory.

The general term $\mu\varepsilon\gamma\varepsilon\theta o\varsigma$ covers line segments, triangles, convex polygones, circles, solids, angles, and arcs of circles. We formalize Euclid's magnitudes of the same kind (line segments being of one kind, triangles being of another, etc.) as an additive semigroup with a total order, $(M, +, <)$, characterized by the following five axioms:

E1 $(\forall x, y)(\exists n \in \mathbb{N})[nx > y]$,

E2 $(\forall x, y)(\exists z)[x < y \Rightarrow x + z = y]$,

E3 $(\forall x, y, z)[x < y \Rightarrow x + z < y + z]$,

E4 $(\forall x)(\forall n \in \mathbb{N})(\exists y)[ny = x]$,

E5 $(\forall x, y, z)(\exists v)[x : y :: z : v]$.

We show that E4 follows from E1-E3, E5; we prove the independence of the axioms E1, E2, E3. We discuss the use of E1 in the proposition V.8; we show that E1 does not follow from the Dedekind completeness axiom (although it does follow from the completeness axiom in an orderd group). We interpret Greek proportion in an Archimedean ordered field, and offer an algebraic interpretation of the axiom *The whole is greater than the part*.

We present schemes of Euclid's propositions; they consist of algebraic formulae representing sequences of (grammatical) sentences, signs representing phrases that occur in the Greek text and references to the axioms, definitions, and other propositions. We discuss under what assumptions these schemes could be turned into modern proofs. Finally, we present algebraic paraphrases of all 25 propositions of book V as derived from the axioms E1-E5.

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Marlena Fila*, Piotr Blaszczyk

Limits of diagrammatic reasoning

We challenge theses of [3] and [4] concerning the Intermediate Value Theorem (IVT); we argue that a diagrammatic reasoning is reliable provided one finds a formula representing the diagram.

IVT states: If $(F, +, \cdot, 0, 1, <)$ is an ordered field, $f : [0, 1] \mapsto F$ is a continuous map such that $f(0)f(1) < 0$, then $f(x) = 0$, for some $x \in (0, 1)$. An accompanying diagram, diag(IVT), depicts a graph of f intersecting a line $(F, <)$, as the function values differ in sign.

(a) In [3], Brown argues that diag(IVT) guarantees the existence of an intersection point.
(b) In [4], Giaquinto argues that diag(IVT) do not guarantee the existence thesis, since continuous functions include non-smooth functions that find no graphic representations.

(ad a) We show that IVT is equivalent to Dedekind Cuts principle (DC): If (A, B) is a Dedekind cut in $(F, <)$, then

$$(\exists! c \in F)(\forall x \in A)(\forall y \in B)[x \leq c \leq y].$$

We also provide a graphic representation for DC.

This equivalence justifies the claim that IVT is as obvious as DC. There is, however, no relation between diag(IVT) and diag(DC), all the more between diag(IVT) and the formula DC. Thus, Brown's claim has to be based on the analytic truth $IVT \Leftrightarrow DC$.

(ad b) Diagrams representing lines $(F, <)$ do not depict whether the field $(F, +, \cdot, 0, 1, <)$ is Euclidean (closed under the square root operation), or $(\mathbb{R}, +, \cdot, 0, 1, <)$, or a real-closed field; graphs of f do not distinguish between polynomial and smooth functions. IVT for polynomials, IVT_p, is valid in real-closed fields (these fields could be *bigger* or *smaller* than real numbers); in fact, IVT_p is the axiom for real-closed fields (next to the Euclidean condition).

Bolzano is believed to give the first proof of IVT. In fact, he sought to prove IVT_p, whilst IVT was just the lemma. Mislead by a diagram, Bolzano proved the theorem not as general as it could be: he proved only that IVT_p is valid in the domain of real numbers.

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Davit Harutyunyan*, Aram Nazaryan, Victor Pambuccian

The Hajja-Martini inequality in a weak absolute geometry

Searching for results that bear some similarity to Propositions 20 and 21 of Book I of Euclid's *Elements*, M. Hajja and H. Martini arrive in [1, Theorem 12] at the following theorem, whose validity they prove in the real Euclidean plane.

Theorem. *Let P be a point in the plane of a triangle ABC. Then there exists a point Q inside or on the boundary of ABC that satisfies.*

$$AQ \leq AP, \ BQ \leq BP, \ CQ \leq CP. \tag{7.1}$$

Aware of the discrepancy between the statement of the theorem, whose notions belong to Hilbert's absolute geometry (whose axioms are the plane axioms of incidence, order, and congruence of groups I, II, and III of Hilbert's *Grundlagen der Geometrie*), which is where one expects a proof to be carried through, and the methods of proof used, the authors ask: "Its fanciful proof, using Zorn's lemma and the Bolzano-Weierstrass theorem, raises the question whether such a heavy machinery is indeed inevitable." [1, p. 13] Moreover, since they can only prove the existence of the point Q, they also ask "whether there is a procedure (an algorithm) to construct the point Q." [1, p. 14] Solving this problem, we prove theorems mentioned below within a very weak plane absolute geometry (all of whose axioms can be deduced inside Hilbert's plane absolute geometry).

Theorem. *For any point P inside or on the boundary of triangle ABC, there is no point Q, different from P, such that Q and P satisfy* (7.1)*.*

Theorem. *For every point P outside of triangle ABC there exists a point Q inside of triangle ABC, such that Q and P satisfy* $(7.1)_<$*.*

In the proof of the last theorem we also provide an algorithm to construct such a point Q.

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Tatyana Ivanova*, Tinko Tinchev

First-order theory of lines in Euclidean plane

The paper [1] gives qualitative spatial reasoning in Euclidean plane based solely on lines. The relations of parallelism and convergence between lines are considered.

In this talk we consider a continuation of [1] by adding a new predicate - perpendicularity. We introduce a first-order theory of lines in Euclidean plane with predicates parallelism, convergence and perpendicularity. The logic is complete with respect to the Euclidean plane, ω - categorical and not categorical in every uncountable cardinality. We prove that the membership problem of the logic is PSPACE-complete.

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Ryszard Mirek

Euclidean Geometry

Renaissance geometry refers directly or indirectly to Euclidean geometry. Fibonacci's *Practica geometriae* written in 1220 contains a large collection of geometry problems arranged into eight chapters with theorems based on Euclid's *Elements*. Piero della Francesca in his treatise solely devoted to the subject of perspective *De Prospectiva Pingendi*, written possibly by about 1474, refers to many Euclid's theorems. For instance in Proposition 1.13, which is known as the first new European theorem in geometry after Fibonacci, the proof refers to the similarity of the triangles. In *Elements* discussion of these issues is included in the Book VI, Proposition 4 to 8. In turn to determine the height of a man one can use the rectangle. The method refers to Euclidean Proposition 16, Book 4, which involves constructing a fifteen-sided figure, equilateral and equiangular. What, however, is the most interesting these and other propositions can be used in the interpretation of the paintings of Piero della Francesca. Luca Pacioli, the pupil of Piero, in his *De divina proportione* moved the mathematical and artistic problems of proportion, especially the mathematics of the golden ratio and its application in architecture.

The purpose of the study is to describe and compare Renaissance geometry in combination with Euclidean one.In the Renaissance the mathematical sciences were in the center of attention and there was a close union between them and the fine arts.

David Pierce

Apollonian proof

This is about a method of proof that seems to have been passed over. Descartes suspected ancient geometers of using algebra, but covering their tracks. However, in the centuries since *La Géométrie* of 1637 [2], persons such as Wallis [7], de Witt [3], and Hamilton [5] who have reworked a basic result of Apollonius [1, I.49–50] have abandoned his visual proof.

Apollonius gives three ways to characterize a conic section: (i) an equation, involving a latus rectum, that we can express in Cartesian form; (ii) the proportion whereby the square on the ordinate varies as the abscissa or abscissas; (iii) an equation of a triangle with a parallelogram or trapezoid. Holding in an affine plane, the latter equation is not usefully translated into the lengths that Descartes has taught us to work with. With the equation, there is a proof-without-words of what today we consider a coordinate change. According to Rosenfeld [6], "Apollonius never mentions parabolic, elliptic, and hyperbolic turns, but no doubt that he used these transformations." The better modern term for what Apollonius uses would be "scissors congruence" or "motivic measure" [4].

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Grzegorz Sitek

Mereological model of arithmetic of natural and real numbers

In [1] the Authors have obtained the full development of Tarski's geometry of solids, that was sketched in [3].*Tarski's Geometry of Solids* is one of the systems of the so called "pointless geometry" and it is based on mereology. In such systems, a notion of a "point" is not assumed as one of the primitive ones. Instead of it, the notion of three-dimensional space and three-dimensional parts of it are accepted. Points are defined then as a special kind of sets of parts of space.

In [2] we have introduced in Tarski's theory the notion of *diameter of mereological ball*. We have shown there, among others, that the set of all diameters together with the relation of *inequality of diameters* is a dense, linearly ordered set, without the least and the greatest element.

Now, we are going to expand these results. We will present a briefly sketch of Tarski's theory and then a sketch of the construction of the notion of natural numbers and real numbers in the theory, together with an interpretation of the operations of addition and multiplication.

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Philippe Balbiani, Tinko Tinchev*

Computability of contact logics with measure

Contact logics [1] are propositional logics interpreted over Boolean contact algebras [3]. They stem from the point-free approaches of geometry put forward by Whitehead. Their language $\mathcal{L}(\leq, C)$ includes Boolean terms representing regions. Let **X** be a set of variables. The set of Boolean terms (s, t, etc) over **X** being denoted $\mathbf{T}(\mathbf{X})$, the set $\mathbf{A}(\mathbf{X})$ of atomic formulas over **X** consists of all expressions of the form $s \leq t$ ("s is part-of t") and $C(s, t)$ ("s is in contact with t"). The set of all formulas (φ, ψ, etc) over **X** is the least set $\mathbf{F}(\mathbf{X})$ containing $\mathbf{A}(\mathbf{X})$ and such that for all $\varphi, \psi \in \mathbf{F}(\mathbf{X})$: $\bot \in \mathbf{F}(\mathbf{X})$, $\neg\varphi \in \mathbf{F}(\mathbf{X})$ and $(\varphi \vee \psi) \in \mathbf{F}(\mathbf{X})$. Of interest are, of course, the sets of all valid formulas determined by the various classes of Boolean contact algebras one may consider. See [1, 5] for detailed investigations.

The combination of topological and size information is a fundamental issue for multifarious applications of spatial reasoning [4]. It can be realized by considering Boolean contact algebras with measure, i.e. algebraic structures (A, C, μ) where (A, C) is a Boolean contact algebra and μ is a positive finite measure on A. Contact logics with measure are extensions of contact logics. Their language $\mathcal{L}(\leq, C, \leq_m)$ contains all additional atomic formulas of the form $s \leq_m t$ ("the size of s is less or equal than the size of t"). Of interest are, again, the sets of all valid formulas determined by the various classes of Boolean contact algebras with measure one may consider.

Using complexity results about linear programming [2], we show that the set of all valid formulas determined by the class of all Boolean contact algebras with measure is in **coNP**. Our proof relies on the equivalence between the satisfiability of a given formula φ and the consistency of an associated system $\mathcal{S}_\varphi$ of linear inequalities. It uses the following facts: the computation of $\mathcal{S}_\varphi$ from φ is possible in non-deterministic polynomial time; if a system of k linear inequalities with integer coefficients of length at most n has a non-negative solution then it has a non-negative solution with at most k positive entries of length in $\mathcal{O}(k.(n + \log k))$.

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8 General

Ryota Akiyoshi*, Andrew Arana

On Gaisi Takeuti's philosophy of mathematics

Gaisi Takeuti (1926-2017) is one of the most distinguished logician in proof theory after Hilbert and Gentzen. He furthered the realization of Hilbert's program by formulating Gentzen's sequent calculus for higher-oder logics, conjecturing the cut-elimination theorem holds for it (Takeuti's conjecture), and obtaining several stunning results in the 1950–60's towards the solution of his conjecture. Though he has been chiefly known as a great mathematician, he wrote many papers in English and Japanese [2, 3, 4] where he expressed his philosophical thoughts.

In this talk, we aim to describe a general outline of our project to investigate Takeuti's philosophy of mathematics. In particular, we point out that there is a crucial difference between Takeuti's program and Hilbert's program, which is based on the fact that Takeuti's philosophical thinking goes back to Nishida's philosophy in Japan.

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James Appleby

Resolving Two Paradoxes About Knowledge States in the Foundations of Intuitionistic Analysis

A choice sequence is a continually growing sequence whose growth may, or may not, be restricted in some way. They were utilised by Brouwer to resolve a crucial issue with his intuitionistic re-foundation of mathematics; specifically, they allowed him to bridge gap between the rationals and the reals.

Choice sequences received no true formalisation in Brouwer's works, however, from [2] onwards, he considered them as a pair of growing objects; a list of elements generated so far, and a list of intensional first order restrictions.

A knowledge state is a formalised way of representing finite information about choice sequences. This allows us to formally represent intensional information about choice sequences, and achieve a notion of choice sequence close to that proposed by Brouwer. The theory $FIM\text{-}KS$ put forward in [1] demonstrates that knowledge states can be used to successfully found intuitionistic analysis. They have also been used in [3] to show that the theory of the creating subject is not needed.

This talk demonstrates that the theory of knowledge states put forward in [1] allows two paradoxes to be derived, and it then outlines their resolution.

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Pavel Arazim

Logical systems as pedagogical and perhaps existential games

The relation between our everyday reasoning and logical systems has become a topic of discussions rich both as far as the quantity of their outputs, as well as the diversirty of opinions is concerned. The straightforward opinion that logic simply captures the rules for correct reasoning as we abide by them gets more and more under attack. Many ways to slacken the relationship, yet still not to lose it completely were undertaken. Still, logical systems are supposed to remain in some sense normative for how we reason. Various versions of logical pluralism have been proposed, recently the idea of reflective equilibrium has been used to give a rationale to logical systems. I propose to see the matters altogether differently.I think logical systems have primarily a pedagogical import, they work as a simulation of correct reasoning. In this way they are very simplified. Just as it is a big difference whether you travel in a spaceship or only train on a simulator. Besides this, logic enables us to have an overview over the basic structures of our reasoning. Normally, we understand that a specific conclusion follows from specific premises, we are immersed in the individual cases. With formal logic, we focus on such general notions as following from. Logic is thus a game which enables us to get a sight of something serious, just in the sense in which Eugen Fink speaks of playing in general as about a form of getting a glance at holistic imports and meanings of the phenomena we encounter in our life. Just a as tragedy in theatre can make us realize something about ourselves, particularly about our emotions, logical systems enable us to learn something about our rationality. In both cases the reality is different and much more complex and perhaps boring.

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Sena Bozdag

A hyperintensional and paraconsistent approach to belief dynamics

I present a new framework that reduces idealizations of reasoning by focusing on information states as the basis of beliefs. I use an extended version of the HYPE model [1] with a preference ordering and a binary belief relation. The model explicitly represents possibly inconsistent and incomplete, non-decreasing collections of information. The static belief operator is a hyperintensional, non-monotonic and paraconsistent modality. The resulting belief sets are consistent but not necessarily closed under logical consequence. On the dynamic aspect, I present two dynamic operators, for belief revision and belief contraction, and their duals. During the process of belief formation and belief change, the agents evaluate the collections of information, rather than the pieces of information or the sets of beliefs. In this way, although there is no apparent distinction between basic or direct information and mere inferences that depend on them, it turns out they behave differently over the course of belief change. As a result, the models are more flexible than HYPE models, and the corresponding propositional logic is weaker than the HYPE logic and the dynamic modal logic is weaker than mainstream logical approaches of belief dynamics.

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Ludovica Conti

One or more Logicisms

The aim of this talk consists in comparing different ways to pursue a logicist project. More in particular, I would compare a proof-theoretic version of logicism, like Tennant's *costructivist logicism* (CL [1]), with two axiomatic versions, namely Heck's *finite Frege Arithmetic* (FFA [2]) and a *free zig-zag logicism* (FZL), obtained by the adoption of a negative free logic and a restricted version of Basic Law V[1].

Both these three systems allows us to derive any instance of the comprehension axiom schema but the different restrictions of the logic (in CL and in FZL) and of the abstraction principles (HP in FFA and BLV in FZL) determine the different strength of the theories.

My two aims consist in, first, discussing the conjecture (proposed by Tennant in [1]) that CL is the intuitionistic (relevant) fragment of Heck's FFA and, secondly, clarifying the existential role of abstraction principles in systems which adopt free logic. Comparing the derivational power of CL and FZL, we can observe that the first one allows us to derive the existential claim $\exists x(x = F)$ only where F is a concept with a finite extension, while the second one allows us to derive also the existential instance of such theorem where F means *natural number* - namely a concept with an infinite extension.

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[1]T-BLV: $\forall F \forall G(\epsilon(F) = \epsilon(G) \leftrightarrow \bigwedge x(Fx \leftrightarrow Gx) \wedge (\phi(F) \wedge \phi(G))$ - where ϕ means "positive" - it contains second-order variables only in the scope of an even number of negation symbols).

Vedran Čačić, Marko Doko, Marko Horvat*

Rearranging absolutely convergent well-ordered series in Banach spaces

Previously at Logic Colloquium, we showed (jointly with Domagoj Vrgoč) that for every absolutely convergent series of real numbers, if its terms are rearranged with respect to any countable ordinal, the newly formed well-ordered series is also absolutely convergent and has the same sum. This time we present an elegant proof of a more general reordering principle in the setting of Banach spaces.

Oguz Korkmaz

Belief as a quantum bit

Belief has been formalized in various frameworks in the course of formal epistemology. Structural differences among these frameworks produce different properties for the concept of belief, one of which is pertinent to the definition of truth values. For example, truth of a belief in classical epistemic modal logic is represented by a classical bit: 0 or 1. Probabilistic approach, on the other hand, assigns propositions to the interval [0,1].

A quantum bit has probabilistic features, so it can be utilized as a representational tool for probabilistic belief. One of the axes on the Bloch sphere can be defined as the axis of probability and another as the axis of information growth. Furthermore, belief updates can theoretically be represented by Bloch sphere rotations which are provided by quantum logic gates.

Harold Hodes

Ramified-types for states of affairs

Assume that for any monadic predication P(u), which predicates the property being P of an object u, there is a unique state-of-affairs (which consists in u being P) which that predication represents; let |P(u)| be thatstate-of-affairs. I will give an argument that for every object u there are distinct properties being P and being Q such that *P(u)* = *Q(u)*. Conisder the following impredicative second-order comprehension principle: (G) some X every y (X(y) iff some Z (y=*Z(u)* and not Z(y))).

So far, no problem. But one might think that states-of-affairs have constituents, and that the following principle of constituency is true for any u and any property being P: (C) The constituents of*P(u)* are exactly u and being P.

By (C), the only constituents of *P(u)* are u and being P, and the only constituents of *Q(u)* are u and being Q, which entails that being P = being Q.

We could reject (C), at least in its full generality. Or we could say that (G) is defective. The former leads to a novel version of logical-atomist metaphysics. The latter points to a (to my knowledge) novel form of ramification.

Alexandre Madeira*, Manisha Jain, Manuel Martins

Towards Invariant bisimulations for parametric multi-valued dynamic logics

Dynamic Logic is a crucial mark on the applied logic to engineering. Despite of its seminal pragmatic role, as the modal logic for the verification of classic programs, its manifest versatility on being adjusted to other computational scenarios (from hybrid systems to quantum computing), stil justifyies further efforts on its application scope expansion.

In order to handle with scenarios where uncertainty is a prime concern, we presented in [1] a method for the systematic construction of many–valued dynamic logics. The method is parametrised by an action lattice, that defines both the computational paradigm and the truth space (corresponding to the underlying Kleene algebra and residuated lattices, respectively). This parametric principle pushed then other theoretical developments, including the a method to the generation of multi-valued epistemic logic, as reported in [2].

This talk contriibutes on a parametric models theory for these logics. It is particular focused in the study of generic (parametric) notions of bisimulation adequate to these formalisms. In analogy of the what we have for standard dynamic logic, we explore, for this generic setting, the usual results like the modal invariance and the Hennessy-Milner correspondences.

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Francisco Martinez Herrera

Justifications and the Lewis argument on ECQ: a relevant note

Taking into account the usual Lewis' independent argument on ECQ, we use Justification Logic as proposed in Artemov [1] [2] to give a simple formalization of the argument. Then, we proceed to clarify the meaning of the classical argument and finally argue that the hypothetical proof-structure shown by means of justification terms provides a clearer, simpler and more explanatory representation of the grounds of some typical arguments to reject ECQ coming from relevant logicians [3, 5, 8].

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Meha Mishra*, A.V. Ravishankar Sarma

An inconsistency tolerant paraconsistent deontic logic of moral conflicts

Moral conflicts are special kind of situations that arise as a reaction to dealing with the conflicting obligations. The resolution of moral conflicts has been studied extensively within the area of moral reasoning whereas representation within the framework of Deontic logic. Despite of moral conflicts are very much part of our linguistic discourse and our tolerance towards them is frequent phenomena, yet the core principles of Standard Deontic Logic fail to capture the intuitive notion of moral conflicts in a satisfactory manner. This poses a major challenge in handling moral conflicts. We argue that situations involving moral conflicts mainly concerned with tolerating inconsistencies, and we assume that best known framework for dealing with moral conflicts are the deontic logics extended with the paraconsistent logic. In paraconsistent logics, a conflict can be represented, operated, isolated, without invalidating the inference rules. I examine three prominent paraconsistent logics; Grahm Priest's logic LP , the logic RM of the school of relevance logic and the Da Costa's logics Cn based on the three valued logic. We emphasize on Deontic paraconsistent logics based on Priest's paraconsisntent logic. I illustrate my work with a classic example from famous Indian epic 'Mahabharata' where the protagonist Arjuna faces moral conflict in the battlefield of Kurukshetra. The inquiry is to find an adequate set of principles to accommodate Arjuna's moral conflict in paraconsistent deontic logics. Meanwhile it is also interesting to relate Krishna's arguments for resolving Arjuna's conflict to paraconsistent approach of conflict tolerance.

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Iaroslav Petik

Reading Feyerabend: from epistemic anarchism to anarchism in foundations of formal systems

Paul Feyerabend was a famous philosopher of science who developed a theory of scientific anarchism. It claims that rationality and scientific method are only the products of one separate tradition of thought which competes with numerous other traditions [Feyerabend]. In this schema science is an eclectic set of different competing systems which functions as an evolving system. But there is no general criterion like rationality for the process of selection. On the other hand the question of foundations of formal systems asks the question what is the ontological foundation for any kind of formal system – mathematics, logic system, the language of programming etc. The program of logicism was aimed at proving that all the chapters of mathematics can be reduced to purely logical constructions. Different philosophical theories in mathematics claim that all the mathematics can be reduced to theory of sets, theory of categories, some constructive principles etc. None of these attempts were eventually successful. The idea of the thesis is to extrapolate and anarchic ideas of Feyerabend on the question about foundations of formal systems. Maybe attempts to find the "main" formal system were all unsuccessful because there is no such system. There is of course the question of practice in the Feyerabend's conception and its counterpart for the case of formal systems. Probably the role of practice should be admitted in this case as well but in a more specified form. In conclusion it should be said that if the anarchism is eligible for the domain of formal systems than the question about foundations of these systems should be also shifted to the study of cooperation of different systems as equal competing structures.

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Tomasz Polacik

Archetypal Rules: Beyond Classical Logic

The notion of archetypal rule was introduced by Lloyd Humberstone, cf. [1]. Informally, we say that a rule r is archetypal for a logic L if, up to provability in L, r is derivable, not invertible and for any other derivable rule s there is a substitution such that the premisses of s are the instances of premisses of r and the conclusion of s is the instance of the conclusion of r. The problem of semantic characterization of archetypal rules in classical propositional logic was solved recently in [4]. Unfortunately, the approach which was applied to classical logic cannot be applied to other logics in a direct way. In this talk we survey some results and shed some light to the general problem of archetypal rules in case of intermediate logics.

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Thomas Macaulay Ferguson, Elisangela Ramirez-Camara*

The Limit of the Strict-Tolerant Hierarchy is Essentially Classical and Even If It's Just LP, That's Probably Okay.

In this paper, we argue that the analyses that tackle the question of whether ST—the non-transitive logic of [2]—is classical logic, offer a framework which is overly restrictive of the notion of metainference. We offer a more elegant and tractable semantics for the strict-tolerant hierarchy based on the three-valued function for the conditional and show how this semantics easily handles the introduction of *mixed* inferences, *i.e.*, inferences involving objects belonging to more than one (meta)inferential level.

We then consider the case of the *deep* ST *theorist*; someone committed to the idea that every level of reasoning follows the bounds-consequence reading offered in [1]. Just as the intuitionist demands constructive metareasoning as well as constructive object language reasoning, the deep ST theorist expects their account of inference to apply to metareasoning. Formally, we extend the translation function that maps ST-valid inferences to LP tautologies so we can account for mixed inferences. While this might seem to reinforce the idea that the deep ST theorist simply endorses LP, we argue instead that it attributes to the deep ST theorist a constancy denied to her by the current analyses of ST metareasoning. Additionally, this account provides a model for Carroll's dialogue between Achilles and the tortoise, with the classical theorist being unable to find a point of difference between themselves and the deep ST theorist, no matter how high in the metatheoretical hierarchy they ascend.

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Luis Estrada-González, Ricardo Nicolás-Francisco*

Negation can be just what it has to

According to Jc Beall [1], there is no logical negation because a logical negation must be either exclusive or exhaustive, but there are no logical reasons that force negation in either way in the correct logic –that, for reasons that we cannot reproduce here, has to be subclassical for Beall (see [2])–. In this paper, we provide some counterarguments to Beall. In particular, we probe characterizations of negation that do not involve the need for exhaustion or exclusion, for example, the flip-flop character of negation as present in Beall's preferred subclassical logic: FDE.

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Luis Estrada-González, Claudia, Lucía Tanús-Pimentel*

Content-sharing in relevant mathematics

The Variable Sharing Principle (VSP) is the necessary condition par excellence in relevance logics to express that antecedent and consequent share content in a valid conditional. Nonetheless, the VSP is but a member of a family of such principles, most of them practically unknown, with distinct degrees of demand and many of them more suitable when dealing with languages with higher expressiveness.

In this paper, we present some such principles and use them to evaluate different results in relevant mathematics, from Meyer's result that any true identity in relevant arithmetic implies any self-identity, to more recent results in inconsistent set theory by Weber and his collaborators.

Vedran Čačić*, Egor V. Kostylev, Juan L. Reutter, Domagoj Vrgoć

Complexity of some fragments of description logics

An important application of modal logic in computer science is the theoretical foundation of *description logic*, which was born out of need to represent knowledge. The important questions of the complexity of a logic, such as the complexity of deciding whether a formula is valid or satisfiable, or the validity of a logical inference, are typically formulated in terms of description logics as the complexity of answering queries. Ontologies, i.e. formalized databases in description logic, are naturally represented by graphs; concepts, i.e. formalized classes of objects correspond to vertices in this context, and roles, i.e. formalized relationships between objects, correspond to edges. Queries can be expressed over concepts or over roles, and as such they correspond to two classes of formulas in the corresponding descriptive logic. In graphs they correspond to searching for vertices or paths with certain properties.

One example, which we intend to present, is the logic $\mathrm{CPDL}^{(\neg)}$, in which it is possible (apart from the usual operators from propositional dynamic logic, like negation, conjunction and disjunction of concepts, and tests, unions, compositions and iterations of programs) to consider the converses of programs (interpreted as inverses of binary relations) and the negations of atomic programs. We know [2] that $\mathrm{PDL}^{(\neg)}$ (i.e. PDL with negations of atomic programs, but with no converses) is EXPTIME-complete, and we believe that an analogous result can be proved for $\mathrm{CPDL}^{(\neg)}$.

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