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Much of the recent research in set theory focuses on singular cardinals and their successors because they are the subjects of both independence proofs and direct ZFC theorems. One of the most famous examples is Easton’s Theorem: The continuum function \( \kappa \mapsto 2^\kappa \) can be any function on the class of regular cardinals as long as it is monotonic and obeys König’s Theorem. It was thought that this result could be extended to singular cardinals, but this was refuted by Silver, who proved that GCH cannot fail for the first time at a singular of uncountable cofinality. The behavior of singular cardinals of uncountable cofinality will be the focus of this talk.

We present recent results in broad strokes. The first, which is joint work with Dima Sinapova, concerns the square property at singularized cardinals. It was known that if \( \kappa \) is inaccessible in an inner model \( V \), and if \( V \subset W \) where \((\kappa^+)^V = (\kappa^+)^W\) and \((\text{cf} \kappa)^W = \omega\), then \( \square_{\kappa, \omega} \) holds in \( W \). However, we find that this does not generalize to the uncountable case: There are models \( V \subset W \) in which \( V \models \\kappa \text{ is inaccessible} \), \((\kappa^+)^V = (\kappa^+)^W\), and \( \omega < (\text{cf} \kappa)^W < \kappa \), and yet \( \square_{\kappa, \tau} \) fails in \( W \) for all \( \tau < \kappa \).

If time allows, we will present a second result, which is joint work with Sy David Friedman, and concerns an Easton-style theorem with regard to the property, “\( \kappa \cap \text{cof}(\omega) \) has a non-reflecting stationary subset”. The class of cardinals \( \kappa \) that satisfy this property can be essentially any class modulo trivial constraints—for example, it is possible to obtain a model in which this property holds when \( \kappa \) is a successor of a regular cardinal, but fails if \( \kappa \) is inaccessible or if \( \kappa \) is the successor of a singular cardinal. The most challenging case here is that of successors of singulars of uncountable cofinality.