EVGENY GORDON, On extension of Haar measure in $\sigma$-compact groups.
Retired, 9 Hanarkis street 31, Ashdod, Israel.
E-mail: gordonevgeny@gmail.com.

In the paper [1] the model of ZFC, where every set of reals, definable by a sequence of ordinals is Lebesgue measurable was constructed under assumptions of existence of an inaccessible cardinal. On the base of this model the model of ZF+DC, in which every set of reals is Lebesgue measurable was presented. In [2] it was proved without the assumption of existence of inaccessible cardinal that the possibility to extend the Lebesgue measure to a non-regular $\sigma$-additive invariant measure defined on all sets of reals is consistent with ZF+DC. Later on Shelah proved that the assumption of existence of inaccessible cardinal cannot be removed from the Solovay's result [3]. In the talk we present the following theorem.

**Theorem 1.** Let $\alpha$ be an arbitrary ordinal definable in ZF. Denote $\text{Base}(X, \beta)$ and $\text{Ext}(X, \beta)$ the statements

1. "$X$ is a $\sigma$-compact group with the base of topology of cardinality $\beta$";
2. "In a $\sigma$-compact group $X$ the left Haar measure can be extended to a left invariant $\sigma$-additive measure defined on all subsets of $X$ definable by a $\beta$-sequence of ordinals".

respectively. Then the following proposition is consistent with ZFC:

$$\forall X \forall \beta < \aleph_\alpha < |\mathbb{R}| \ (\text{Base}(X, \beta) \rightarrow \text{Ext}(X, \beta))$$