► EVGENY GORDON, *On extension of Haar measure in σ-compact groups.*
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In the paper [1] the model of ZFC, where every set of reals, definable by a sequence of ordinals is Lebesgue measurable was constructed under assumptions of existence of an inaccessible cardinal. On the base of this model the model of ZF+DC, in which every set of reals is Lebesgue measurable was presented. In [2] it was proved without the assumption of existence of inaccessible cardinal that the possibility to extend the Lebesgue measure to a non-regular σ-additive invariant measure defined on all sets of reals is consistent with ZF+DC. Later on Shelah proved that the assumption of existence of inaccessible cardinal cannot be removed from the Solovay's result [3]. In the talk we present the following theorem.

THEOREM 1. *Let α be an arbitrary ordinal definable in ZF. Denote $Base(X, \beta)$ and $Ext(X, \beta)$ the statements*

1. *"X is a σ-compact group with the base of topology of cardinality β";*
2. *"In a σ-compact group X the left Haar measure can be extended to a left invariant σ-additive measure defined on all subsets of X definable by a β-sequence of ordinals".*

respectively. Then the following proposition is consistent with ZFC:

$$\forall X \forall \beta < \aleph_\alpha < |\mathbb{R}| \ \ (Base(X, \beta) \longrightarrow Ext(X, \beta))$$

[1] ROBERT SOLOVAY, *A model of set theory in which every set of reals is Lebesgue measurable*, ***Annals of Mathematics***, vol. 142 (1969), no. 2, pp. 381–420.

[2] GERALD SAKS, *Measure-theoretical uniformity in recursion theory and set theory*, ***Transactions of the American Mathematical Society***, vol. 48 (1984), no. 1, pp. 1–47.

[3] SAHARON SHELAH, *Can you take Solovay's inaccessible away?*, ***Israel Journal of Mathematics***, vol. 48 (1984), no. 1, pp. 1–47.