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We study a family of axioms expressing

$$(*) \qquad \text{"All axioms of PA are true."}$$

where PA denotes Peano Arithmetic. More precisely, each such axiom states that all axioms *from a chosen axiomatization of* PA are true.

We start with a very natural theory of truth $\mathrm{CT}^-(\mathrm{PA})$ which is a finite extension of PA in the language of arithmetic augmented with a fresh predicate $T$ to serve as a truth predicate *for the language of arithmetic*. Additional axioms of this theory are straightforward translations of inductive Tarski truth conditions. To study various possible ways of expressing $(*)$, we investigate extensions of $\mathrm{CT}^-(\mathrm{PA})$ with axioms of the form

$$(**) \qquad \forall x \ \big(\delta(x) \rightarrow T(x)\big).$$

In the above (and throughout the whole abstract) $\delta(x)$ is an arithmetical $\Delta_0$ formula which is proof-theoretically equivalent to the standard axiomatization of PA with the induction scheme, i.e. the equivalence

$$\forall x \big(\mathrm{Prov}_{\delta}(x) \equiv \mathrm{Prov}_{\mathrm{PA}}(x)\big).$$

is provable in $I\Sigma_1$. For every such $\delta$, the extension of $\mathrm{CT}^-(\mathrm{PA})$ with axiom $(**)$ will be denoted $\mathrm{CT}^-[\delta]$.

In particular we are interested in the arithmetical strength of theories $\mathrm{CT}^-[\delta]$. The "line" demarcating extensions of $\mathrm{CT}^-(\mathrm{PA})$ which are conservative over PA from the nonconservative ones is known in the literature as the *Tarski Boundary*. So far, there seemed to be the least (in terms of deductive strength) natural extension of $\mathrm{CT}^-(\mathrm{PA})$ on the nonconservative side of the boundary, whose one axiomatization is given by $\mathrm{CT}^-(\mathrm{PA})$ and $\Delta_0$ induction for the extended language (the theory is called $\mathrm{CT}_0$). In contrast to this, we prove the following result:

Theorem 1. *For every r.e. theory* Th *in the language of arithmetic the following are equivalent:*

1. $\mathrm{CT}_0 \vdash \mathrm{Th}$
2. *there exists* $\delta$ *such that* $\mathrm{CT}^-[\delta]$ *and* Th *have the same arithmetical consequences.*

Secondly, we use theories $\mathrm{CT}^-[\delta]$ to measure the distance between $\mathrm{CT}^-(\mathrm{PA})$ and the Tarski Boundary. We prove

Theorem 2. *There exists a family* $\{\delta_f\}_{f\in\omega^{<\omega}}$ *such that*

1. *for every* $f \in \omega^{<\omega}$, $\mathrm{CT}^-[\delta_f]$ *is conservative over* PA;
2. *if* $f \subsetneq g$, *then* $\mathrm{CT}^-[\delta_g]$ *properly extends* $\mathrm{CT}^-[\delta_f]$;
3. *if* $f \perp g$ *then* $\mathrm{CT}^-[\delta_g] \cup \mathrm{CT}^-[\delta_f]$ *is nonconservative over* PA.