▸ GUILLERMO BADIA, PETR CINTULA, AND ANDREW TEDDER, *How much propositional logic suffices for Rosser's undecidability theorem?*.
Institute of Computer Science of the Czech Academy of Sciences
Pod Vodárenskou věží 271/2, Prague 8, 182 07, Czech Republic.
E-mail: ajtedder.at@gmail.com.
E-mail: cintula@cs.cas.cz.
School of Historical and Philosophical Inquiry, University of Queensland, Australia.
E-mail: guillebadia89@gmail.com.

Rosser [3] famously established that Peano Arithmetic was essentially undecidable against the background of classical first-order logic, that no consistent extension of that theory against that logic was decidable, and this result was extended to Robinson Arithmetic in [4]. We extend the result further by considering not only a weaker arithmetic theory, but a weaker *propositional* logic governing the behaviour of logical connectives. Following Hájek's paper [2] and his later unpublished work on fuzzy arithmetics, we prove essential undecidability for a version of Robinson arithmetic, with predicates (rather than functions) to interpret the arithmetic operations, against the background of a propositional logic with very few logical assumptions. Our logic is much weaker thank the one used by Hájek; it is a variant of Brady's **BBQ** [1] with weakening, in a language including falsum (in terms of which negation is defined), and a crisp (i.e., two-valued) identity predicate. The key logical fact is that, in this system, one can regain enough of the logical inferential machinery to establish the necessary arithmetic facts, the key fact being that all the theory-specific predicates can be shown to be crisp when applied to *numerals*. Therefore, our result suggests that essential undecidability is a property largely dependent on the arithmetic theory, rather than the background propositional logic.

[1] R.T. Brady. *A content semantics for quantified relevant logics. I* ***Studia Logica***, vol. 47.2 (1988), pp. 111-127.

[2] P. Hájek, *Mathematical Fuzzy Logic and Natural Numbers*, ***Fundamenta Informaticae***, vol. 81 (2007), pp. 155–163.

[3] B. Rosser, *Extensions of some theorems of Gödel and Church*, ***Journal of Symbolic Logic***, vol. 1 (1936), no. 3, pp. 87–91.

[4] A. Tarski, A. Mostowski, R.M. Robinson, ***Undecidable Theories***, North-Holland, 1953.