We identify natural theorems of higher-order arithmetic that are independent of the medium range of the G"odel hierarchy ([7]); this range includes most sub-systems of second-order arithmetic. We then obtain a number of independent hierarchies that are parallel to the medium range:

1. The compactness hierarchy based on Cousin’s lemma ([1], 1895).
2. The Lindel"of hierarchy based on Lindel"of’s lemma ([2], 1903).
3. The local-global hierarchy based on Pincherle’s theorem ([5,6], 1882).
4. The first net hierarchy based on the monotone convergence theorem for nets, aka Moore-Smith sequences ([3], 1922).
5. The second net hierarchy based on moduli of convergence for nets.
6. The neighbourhood function hierarchy based on NFP from [4].
7. Variations of these hierarchies.

We work with the G"odel hierarchy based on inclusion and higher-order rather than second-order systems.

This research is part of my joint project with Dag Normann on the Reverse Mathematics and computability theory of the uncountable (see [4] for an introduction).

REFERENCES.