

- SAM SANDERS, *Umpteen parallel hierarchies and the Gödel hierarchy*.

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We identify natural theorems of higher-order arithmetic that are independent of the medium range of the *Gödel hierarchy* ([7]); this range includes most sub-systems of second-order arithmetic. We then obtain a number of independent hierarchies that are parallel to the medium range:

1. The *compactness* hierarchy based on *Cousin's lemma* ([1], 1895).
2. The *Lindelöf* hierarchy based on *Lindelöf's lemma* ([2], 1903).
3. The *local-global* hierarchy based on *Pincherle's theorem* ([5, 6], 1882).
4. The first *net* hierarchy based on the monotone convergence theorem for *nets*, aka Moore-Smith sequences ([3], 1922).
5. The second *net* hierarchy based on moduli of convergence for nets.
6. The *neighbourhood function* hierarchy based on NFP from [4].
7. Variations of these hierarchies.

We work with the Gödel hierarchy based on inclusion and higher-order rather than second-order systems.

This research is part of my joint project with Dag Normann on the Reverse Mathematics and computability theory of the uncountable (see [4] for an introduction).

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