

- SAM SANDERS, *Ump teen parallel hierarchies and the Gödel hierarchy.*

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We identify natural theorems of higher-order arithmetic that are independent of the medium range of the *Gödel hierarchy* ([7]); this range includes most sub-systems of second-order arithmetic. We then obtain a number of independent hierarchies that are parallel to the medium range:

1. The *compactness* hierarchy based on *Cousin's lemma* ([1], 1895).
2. The *Lindelöf* hierarchy based on *Lindelöf's lemma* ([2], 1903).
3. The *local-global* hierarchy based on *Pincherle's theorem* ([5, 6], 1882).
4. The first *net* hierarchy based on the monotone convergence theorem for *nets*, aka Moore-Smith sequences ([3], 1922).
5. The second *net* hierarchy based on moduli of convergence for nets.
6. The *neighbourhood function* hierarchy based on NFP from [4].
7. Variations of these hierarchies.

We work with the Gödel hierarchy based on inclusion and higher-order rather than second-order systems.

This research is part of my joint project with Dag Normann on the Reverse Mathematics and computability theory of the uncountable (see [4] for an introduction).

REFERENCES.

- [1] Pierre Cousin, *Sur les fonctions de n variables complexes*, Acta Math. **19** (1895), 1–61.
- [2] Ernst Lindelöf, *Sur Quelques Points De La Théorie Des Ensembles*, Comptes Rendus (1903), 697–700.
- [3] E. H. Moore and H. Smith, *A General Theory of Limits*, Amer. J. Math. **44** (1922), 102–121.
- [4] Dag Normann and Sam Sanders, *On the mathematical and foundational significance of the uncountable*, Journal of Mathematical Logic, <https://doi.org/10.1142/S0219061319500016> (2018).
- [5] ———, *Pincherle's theorem in Reverse Mathematics and computability theory*, Submitted, arXiv: <https://arxiv.org/abs/1808.09783> (2018).
- [6] Salvatore Pincherle, *Sopra alcuni sviluppi in serie per funzioni analitiche* (1882), Opere Scelte, I, Roma (1954), 64–91.
- [7] Stephen G. Simpson, *The Gödel hierarchy and reverse mathematics.*, Kurt Gödel. Essays for his centennial, 2010, pp. 109–127.