It seems at least unlikely that there is a natural way to extend the \{0, 1\}-interpretations of PL to arbitrary values from the unit interval \([0, 1]\) := \(\{x \in \mathbb{R} : 0 \leq x \leq 1\}\).

For technical reasons, we use the following PL-language: \(p_1, p_2, p_3, \ldots, \neg \phi, \bigwedge \Phi, \bigvee \Psi\), where \(\Phi, \Psi\) stand for finite sets of formulæ previously built. For instance, \(\phi \land \psi := \bigwedge \{\phi, \psi\}\) apv(\(\phi\)) be the set of atomic propositional variables \(p\) occurring in \(\phi\).

Formulæ are binarily interpreted in well-known way, e. g. \(V(p) \in \{0, 1\}\), \(V(\neg \phi) = 1\) iff \(V(\phi) = 0\), \(V(\bigwedge \Phi) = 1\) iff \(V(\phi) = 1\) for all \(\phi \in \Phi\). \(V(\bigvee \emptyset) = 0\).

I. Let \(\text{apv}(\phi) = \{p^1, \ldots, p^n\}\). Call \(\kappa := \bigwedge \Lambda\) fundamental iff either \(p^k \in \Lambda\) or \(\neg p^k \in \Lambda\) for every \(1 \leq k \leq n\), \(\kappa_1 \perp \kappa_2\) if \(\Lambda_1 \neq \Lambda_2\). There is an unique set \(K\) of fundamental \(\kappa\) such that \(\models \phi \leftrightarrow \bigvee K\), where \(\delta := \bigvee K\) corresponds to (full) DNF.

Now let \(W(p^k) \in [0, 1]\) be any multi-valued assignment. The MVI or Buchholz valuation \(W'\) is constructed as follows:

- \(W'(p^k) := W(p^k)\),
- \(W'(\neg p^k) := 1 - W(p^k)\),
- \(W'(\kappa) := \prod_{\lambda \in \Lambda} W'(\lambda)\),
- \(W'(\delta) := \sum_{\kappa \in K} W'(\kappa)\),
- \(W'(\phi) := W'(\delta)\);

revealing the somehow evident principles used.

Let \(W_5 := \frac{1}{2}\). \(W_5^\pi\) has been paradigm for MVI, whereby PL-probability \(\pi_{\text{PL}}(\phi)\) equals the number of rows with value 1 in the (binary) truth table of \(\phi\) divided by \(2^n\)—originating with the Tractatus, probably.

II. Except for \(\pi(\bigwedge \Lambda) = \prod_{\lambda \in \Lambda} \pi(\lambda)\), the principles used to define \(W'\) are all properties of probability functions \(\pi\) in the sense of probability logic—maybe of many-valued logic at all—, and we showed that the literal-independent \(\pi\) can be identified with our \(W'\).

However, the nearness to stochastics is deceptive: Consider, e. g., coin toss, where \(\pi(\text{head} \land \text{tail}) \neq \pi(\text{head}) \cdot \pi(\text{tail})\).

III. What have we found?

Acknowledgment. After a joint quest, Wilfried Buchholz had solved the problem technically.—Preliminary versions were presented at ASL-APA and UniLog 2018. Thanks to many participants, Walter Carnielli, Luis Estrada-González, Peter Maer-Borst, Schafag Kerimova, Andreas Haltenhoff.