▸ JOACHIM MUELLER-THEYS, *Multi-valued interpretations.*
Kurpfalzstr. 53, 69 226 Nußloch bei Heidelberg, Germany.
E-mail: `mueller-theys@gmx.de`.

It seems at least unlikely that there is an natural way to extend the $\{0,1\}$-interpretations of PL to arbitrary values from the unit interval $[0,1] := \{x \in \mathrm{IR}\colon 0 \leq x \leq 1\}$.

0. For technical reasons, we use the following PL-language: $p_1, p_2, p_3, \dots$, $\neg\phi$, $\bigwedge\Phi$, $\bigvee\Psi$, where Φ, Ψ stand for *finite* sets of formulæ previously built. For instance, $\phi \wedge \psi := \bigwedge\{\phi, \psi\}$. *apv*$(\phi)$ be the set of atomic propositional variables p occurring in ϕ.

Formulæ are binarily interpreted in well-known way, e. g. $V(p) \in \{0,1\}$, $V'(\neg\phi) = 1$ iff $V'(\phi) = 0$, $V'(\bigwedge\Phi) = 1$ iff $V'(\phi) = 1$ for all $\phi \in \Phi$. $V'(\bigvee\emptyset) = 0$. $\models \phi$:iff $V'(\phi) = 1$ for all V, whence $\models \phi \leftrightarrow \psi$ iff $V'(\phi) = V'(\psi)$.

I. Let $\mathrm{apv}(\phi) = \{p^1, \dots, p^n\}$. Call $\kappa := \bigwedge\Lambda$ *fundamental* iff either $p^k \in \Lambda$ or $\neg p^k \in \Lambda$ for every $1 \leq k \leq n$. $\kappa_1 \perp \kappa_2$ if $\Lambda_1 \neq \Lambda_2$. There is an unique set K of fundamental κ such that $\models \phi \leftrightarrow \bigvee K$, where $\delta := \bigvee K$ corresponds to (full) DNF.

Now let $W(p^k) \in [0,1]$ be any *multi-valued assignment*. The *MVI* or *Buchholz valuation* W' is constructed as follows:
$W'(p^k) := W(p^k)$,
$W'(\neg p^k) := 1 - W'(p^k)$,
$W'(\kappa) := \prod_{\lambda\in\Lambda} W'(\lambda)$,
$W'(\delta) := \sum_{\kappa\in K} W'(\kappa)$,
$W'(\phi) := W'(\delta)$;
revealing the somehow evident principles used.

Let $W_{.5} := \frac{1}{2}$. $W'_{.5} = \pi_{\mathrm{PL}}$ has been *paradigm* for MVI, whereby *PL-probability* $\pi_{\mathrm{PL}}(\phi)$ equals the number of rows with value 1 in the (binary) truth table of ϕ divided by 2^n—originating with the *Tractatus*, probably.

II. Except for $\pi(\bigwedge\Lambda) = \prod_{\lambda\in\Lambda} \pi(\lambda)$, the principles used to define W' are all properties of probability functions π in the sense of probability logic—maybe of many-valued logic at all—, and we showed that the literal-independent π can be identified with our W'.

However, the nearness to stochastics is deceptive: Consider, e. g., coin toss, where $\pi(\mathrm{head} \wedge \mathrm{tail}) \neq \pi(\mathrm{head}) \cdot \pi(\mathrm{tail})$.

III. What have we found?

Acknowledgment. After a joint quest, Wilfried Buchholz had solved the problem technically.—Preliminary versions were presented at ASL-APA and UniLog 2018. Thanks to many participants, Walter Carnielli, Luis Estrada-González, Peter Maier-Borst, Schafag Kerimova, Andreas Haltenhoff.