Class $\mathcal{K}$ of finite $L$-structures has the extension property for partial automorphisms (EPPA or Hrushovski property) if for every structure $A \in \mathcal{K}$ there exists a structure $B \in \mathcal{K}$ such that $A \subseteq B$ and every partial automorphism of $A$ (that is, an isomorphism of its substructures) extends to automorphism of $B$.

It was shown by Hrushovski in 1992 that the class of all finite graphs has EPPA. Thanks to numerous applications in group theory and topological dynamics the search for more classes with EPPA continues since then. The Herwig–Lascar theorem, a deep result in the area, provides a structural condition for a class to have EPPA and has been used to prove EPPA for most known examples.

Recently a new link to the structural Ramsey theory has been established. The condition given by the Herwig–Lascar theorem is almost identical to a condition given in [3] for the existence of a precompact Ramsey expansion. In [1], a class $\mathcal{C}_F$ (constructed using the Hrushovski predimension construction) is studied, giving a counterexample to questions in Ramsey theory. It is shown that $\mathcal{C}_F$ has no EPPA, however, it is conjectured that a certain expansion (adding an orientation of edges) has EPPA.

We prove this conjecture using a new strengthening of the Herwig–Lascar theorem [2]. Because $\mathcal{C}_F$ has non-unary algebraic closures, new techniques needs to be developed. In particular, we work with a category of structures in languages equiped with a permutation group on the symbols. We discuss these new techniques and their applications.