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A Complete Intuitionistic Temporal Logic for Topological Dynamics.

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Linear temporal logic (LTL) is a poly-modal propositional logic which allows for the representation of various tenses including $\circ$ ('next') and $\diamond$ ('eventually'), and *dynamical (topological) systems* are pairs (X, S) consisting of the action of a continuous function $S\colon X \to X$ on the topological space X. Dynamical systems naturally provide semantics for the language of LTL by using the function S to interpret $\circ$, $\diamond$ and the topological structure to interpret implication, thus giving rise to an intuitionistic variant of linear temporal logic. Under this interpretation, it is natural to enrich the language of LTL with a universal modality, $\forall$.

In our talk we will show how this language is expressive enough to capture non-trivial phenomena such as Poincaré recurrence and minimality. We will then introduce a 'minimal' axiomatization $\mathsf{ITL}^0_{\diamond\forall}$ for intuitionistic temporal logic and discuss a few (in)completeness results:

1. The logic $\mathsf{ITL}^0_{\diamond\forall}$ with tenses $\circ, \diamond, \forall$ is sound and complete for
   (a) the class of all dynamical systems, and
   (b) the set of all dynamical systems based on the rational numbers, $\mathbb{Q}$.
   In contrast, $\mathsf{ITL}^0_{\diamond\forall}$ is not complete for interpretations based on $\mathbb{R}^n$.
2. The $\forall$-free fragment $\mathsf{ITL}^0_{\diamond}$ is complete for
   (a) the class of all finite dynamic topological systems,
   (b) the class of dynamical systems based over $\mathbb{R}^n$ for any fixed $n \geq 2$, and
   (c) the class of dynamical systems based on the Cantor space.
   However, $\mathsf{ITL}^0_{\diamond}$ is incomplete for the real line.

Finally, we show that $\Box$ ('henceforth') is not definable in terms of $\diamond$ and discuss some problems and possible approaches to including $\Box$ in our logic.