Our main interested objects are PAC structures ([5, Definition 3.1]), which generalize perfect PAC fields. We show that the first order theories of PAC structures are determined by their Galois groups. Really, in joint work [3] with Dobrowolski, we showed that if given PAC structures have Galois groups isomorphic over a Galois group of a common substructure, then they are elementary equivalent, which generalizes the elementary equivalence theorem for PAC fields (see [4, Theorem 20.3.3]).

In the sequel work [6], we try to generalize criterion to say the theory of a PAC field is $NSOP_n$ if the theory of the complete system of its Galois group is $NSOP_n$ for $n \geq 1$ ([1, Theorem 3.9] and [7, Corollary 7.2.7, Proposition 7.2.8]). Zoé’s amalgamation theorem with respect to complete systems ([1, Theorem 3.1]) is crucial in this criterion.

To generalize this amalgamation theorem, we introduce notions of sorted Galois groups and of sorted complete system of sorted Galois groups. Using sorted complete systems, we generalize Zoé’s amalgamation theorem to PAC structures. We also generalize the criterion of $NSOP_n$ for PAC fields to PAC structures.