We study a nonstandard formulation for time-dependent partial differential equations of the form

\[
\begin{align*}
    u_t &= f(u, P_1(u), \ldots, P_n(u)) \quad \text{in } \Omega \subseteq \mathbb{R}^k \\
    u(0) &= u_0
\end{align*}
\]

with \( P_i(u) = \sum_{\alpha \in I_i} a_\alpha \frac{\partial^{|\alpha|} u}{\partial x^\alpha} \) and with distributional or measure-valued initial data \( u_0 \).

Equations of this form include linear and nonlinear diffusion and systems of conservation laws. Despite their similarities, many of these problems are studied with different techniques and have different notions of solutions [1, 2, 3].

Working in the setting of Robinson’s nonstandard analysis, we discretize the differential operators \( P_i \) in space by means of finite differences of an infinitesimal step \( \varepsilon \): the resulting hyperfinite system of ODEs is formally equivalent to (1). If \( f \) is Lipschitz continuous, this system has a unique solution that induces a standard solution to problem (1). For the forward-backward heat equations, this standard solution coincides with the solution obtained with a vanishing viscosity approach; moreover, it is possible to characterize its asymptotic behaviour [1].

We suggest that this nonstandard formulation could be successfully employed in the study of other classes of problems and might lead to novel qualitative information about their solutions.

