DMITRY EMELYANOV, BEIBUT KULPESHOV, SERGEY SUDOPLATOV, On compositions of structures and compositions of theories.
Novosibirsk State Technical University, Novosibirsk, Russia.
E-mail: dima-pavllyk@mail.ru.
International Information Technology University, Almaty, Kazakhstan.
E-mail: b.kulpeshov@iitu.kz.
Sobolev Institute of Mathematics, Novosibirsk State Technical University, Novosibirsk, Russia.
E-mail: sudoplat@math.nsc.ru.

We consider both compositions of structures and compositions of theories and apply these compositions obtaining compositions of algebras of binary formulas [1].

Let $\mathcal{M}$ and $\mathcal{N}$ be structures of relational languages $\Sigma_\mathcal{M}$ and $\Sigma_\mathcal{N}$, respectively. We define the composition $\mathcal{M}[\mathcal{N}]$ of $\mathcal{M}$ and $\mathcal{N}$ satisfying $\Sigma_{\mathcal{M}[\mathcal{N}]} = \Sigma_\mathcal{M} \cup \Sigma_\mathcal{N}$, $\mathcal{M}[\mathcal{N}] = M \times N$ and the following conditions:

1) if $R \in \Sigma_\mathcal{M} \setminus \Sigma_\mathcal{N}$, then $\mu(R) = n$, then $\{(a_1, b_1), \ldots, (a_n, b_n)\} \in R_{\mathcal{M}[\mathcal{N}]}$ if and only if $(a_1, \ldots, a_n) \in R_M$.
2) if $R \in \Sigma_\mathcal{N} \setminus \Sigma_\mathcal{M}$, then $\mu(R) = n$, then $\{(a_1, b_1), \ldots, (a_n, b_n)\} \in R_{\mathcal{M}[\mathcal{N}]}$ if and only if $a_1 = \ldots = a_n$ and $(b_1, \ldots, b_n) \in R_N$.
3) if $R \in \Sigma_\mathcal{M} \cap \Sigma_\mathcal{N}$, then $\mu(R) = n$, then $\{(a_1, b_1), \ldots, (a_n, b_n)\} \in R_{\mathcal{M}[\mathcal{N}]}$ if and only if $(a_1, \ldots, a_n) \in R_M$ or $a_1 = \ldots = a_n$ and $(b_1, \ldots, b_n) \in R_N$.

The composition $T = \text{Th}(\mathcal{M}[\mathcal{N}])$ is called the composition $T_1[T_2]$ of the theories $T_1 = \text{Th}(\mathcal{M})$ and $T_2 = \text{Th}(\mathcal{N})$.

**Theorem 1.** If $\mathcal{M}$ and $\mathcal{N}$ have transitive automorphism groups then $\mathcal{M}[\mathcal{N}]$ has a transitive automorphism group, too.

By Theorem 1, $T = \text{Th}(\mathcal{M}[\mathcal{N}])$ is transitive, and the operation $\mathcal{M}[\mathcal{N}]$ can be considered as a variant of transitive arrangements of structures [2].

The composition $\mathcal{M}[\mathcal{N}]$ is called $E$-definable if $\mathcal{M}[\mathcal{N}]$ has an $E$-definable equivalence relation $E$ whose $E$-classes are universes of the copies of $\mathcal{N}$ forming $\mathcal{M}[\mathcal{N}]$. By the definition, each $E$-definable composition $\mathcal{M}[\mathcal{N}]$ is represented as an $E$-combination [3] of copies of $\mathcal{N}$ with an extra-structure generated by predicates on $\mathcal{M}$ and linking elements of the copies of $\mathcal{N}$.

**Theorem 2.** If a composition $\mathcal{M}[\mathcal{N}]$ is $E$-definable then the theory $\text{Th}(\mathcal{M}[\mathcal{N}])$ uniquely defines the theories $\text{Th}(\mathcal{M})$ and $\text{Th}(\mathcal{N})$, and vice versa.

**Theorem 3.** If a composition $\mathcal{M}[\mathcal{N}]$ is $E$-definable then the algebra $\Psi_T$ of binary isolating formulas for $T = \text{Th}(\mathcal{M}[\mathcal{N}])$ is isomorphic to the composition $\Psi_{T_1}[\Psi_{T_2}]$ of the algebras $\Psi_{T_1}$ and $\Psi_{T_2}$ of binary isolating formulas for $T_1 = \text{Th}(\mathcal{M})$ and $T_2 = \text{Th}(\mathcal{N})$.