▸ DMITRY EMELYANOV, BEIBUT KULPESHOV, SERGEY SUDOPLATOV, *On compositions of structures and compositions of theories.*
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We consider both compositions of structures and compositions of theories and apply these compositions obtaining compositions of algebras of binary formulas [1].

Let $\mathcal{M}$ and $\mathcal{N}$ be structures of relational languages $\Sigma_{\mathcal{M}}$ and $\Sigma_{\mathcal{N}}$, respectively. We define the composition $\mathcal{M}[\mathcal{N}]$ of $\mathcal{M}$ and $\mathcal{N}$ satisfying $\Sigma_{\mathcal{M}[\mathcal{N}]}=\Sigma_{\mathcal{M}}\cup\Sigma_{\mathcal{N}}$, $M[N]=M\times N$ and the following conditions:

1) if $R\in\Sigma_{\mathcal{M}}\setminus\Sigma_{\mathcal{N}}$, $\mu(R)=n$, then $((a_1,b_1),\ldots,(a_n,b_n))\in R_{\mathcal{M}[\mathcal{N}]}$ if and only if $(a_1,\ldots,a_n)\in R_{\mathcal{M}}$;

2) if $R\in\Sigma_{\mathcal{N}}\setminus\Sigma_{\mathcal{M}}$, $\mu(R)=n$, then $((a_1,b_1),\ldots,(a_n,b_n))\in R_{\mathcal{M}[\mathcal{N}]}$ if and only if $a_1=\ldots=a_n$ and $(b_1,\ldots,b_n)\in R_{\mathcal{N}}$;

3) if $R\in\Sigma_{\mathcal{M}}\cap\Sigma_{\mathcal{N}}$, $\mu(R)=n$, then $((a_1,b_1),\ldots,(a_n,b_n))\in R_{\mathcal{M}[\mathcal{N}]}$ if and only if $(a_1,\ldots,a_n)\in R_{\mathcal{M}}$, or $a_1=\ldots=a_n$ and $(b_1,\ldots,b_n)\in R_{\mathcal{N}}$.

The theory $T=\mathrm{Th}(\mathcal{M}[\mathcal{N}])$ is called the *composition* $T_1[T_2]$ of the theories $T_1=\mathrm{Th}(\mathcal{M})$ and $T_2=\mathrm{Th}(\mathcal{N})$.

Theorem 1. *If $\mathcal{M}$ and $\mathcal{N}$ have transitive automorphism groups then $\mathcal{M}[\mathcal{N}]$ has a transitive automorphism group, too.*

By Theorem 1, $T=\mathrm{Th}(\mathcal{M}[\mathcal{N}])$ is transitive, and the operation $\mathcal{M}[\mathcal{N}]$ can be considered as a variant of transitive arrangements of structures [2].

The composition $\mathcal{M}[\mathcal{N}]$ is called *E-definable* if $\mathcal{M}[\mathcal{N}]$ has an $\emptyset$-definable equivalence relation E whose E-classes are universes of the copies of $\mathcal{N}$ forming $\mathcal{M}[\mathcal{N}]$. By the definition, each E-definable composition $\mathcal{M}[\mathcal{N}]$ is represented as a E-combination [3] of copies of $\mathcal{N}$ with an extra-structure generated by predicates on $\mathcal{M}$ and linking elements of the copies of $\mathcal{N}$.

Theorem 2. *If a composition $\mathcal{M}[\mathcal{N}]$ is E-definable then the theory* $\mathrm{Th}(\mathcal{M}[\mathcal{N}])$ *uniquely defines the theories* $\mathrm{Th}(\mathcal{M})$ *and* $\mathrm{Th}(\mathcal{N})$*, and vice versa.*

Theorem 3. *If a composition $\mathcal{M}[\mathcal{N}]$ is E-definable then the algebra $\mathfrak{P}_T$ of binary isolating formulas for $T=\mathrm{Th}(\mathcal{M}[\mathcal{N}])$ is isomorphic to the composition $\mathfrak{P}_{T_1}[\mathfrak{P}_{T_2}]$ of the algebras $\mathfrak{P}_{T_1}$ and $\mathfrak{P}_{T_2}$ of binary isolating formulas for $T_1=\mathrm{Th}(\mathcal{M})$ and $T_2=\mathrm{Th}(\mathcal{N})$.*

[1] I.V. Shulepov, S.V. Sudoplatov, *Algebras of distributions for isolating formulas of a complete theory*, ***Siberian Electronic Mathematical Reports***, Vol. 11 (2014), pp. 362–389.

[2] S.V. Sudoplatov, *Transitive arrangements of algebraic systems*, ***Siberian Mathematical Journal***, Vol. 40, Issue 6 (1999), pp. 1142–1145.

[3] S.V. Sudoplatov, *Combinations of structures*, ***The Bulletin of Irkutsk State University. Series "Mathematics"***, Vol. 24 (2018), pp. 65–84.