▸ SERGEY DAVIDOV, SENIK ALVRTSYAN, DAVIT SHAHNAZARYAN, *Invertible binary algebras principally isotopic to a group.*
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A binary groupoid $Q(A)$ is a non-empty set Q together with a binary operation A. Binary groupoid $Q(A)$ is called quasigroup if for all ordered pairs $(a, b) \in Q^2$ exists unique solutions $x, y \in Q$ of the equations $A(a, x) = b$ and $A(y, a) = b$. The solutions of these equations will be denoted by $x = A^{-1}(a, b)$ and $y = {}^{-1}A(b, a)$, respectively. A binary algebra $(Q; \Sigma)$ is called invertible algebra or system of quasigroups if each operation in Σ is a quasigroup operation.

We obtained characterizations of invertible algebras isotopic to a group or an abelian group by the second-order formula.

Definition 1. We say that a binary algebra $(Q; \Sigma)$ is isotopic to the groupoid $Q(\cdot)$, if each operation in Σ is isotopic to the groupoid $Q(\cdot)$, i.e. for every operation $A \in \Sigma$ there exists permutations $\alpha_A, \beta_A, \gamma_A$ of Q, that:

$$\gamma_A A(x, y) = \alpha_A x \cdot \beta_A y,$$

for every $x, y \in Q$. Isopoty is called principal if $\gamma_A = epsilon(\epsilon$ - unit permutation) for every $A \in \Sigma$.

Theorem 2. *The invertible algebra $(Q; \Sigma)$ is a principally isotopic to the abelian group, if and only if the following second-order formula*

$$A({}^{-1}A(B(x, B^{-1}(y, z)), u), v) = B(x, B^{-1}(y, A({}^{-1}A(z, u), v))),$$

is valid in the algebra $(Q; \Sigma \cup \Sigma^{-1} \cup {}^{-1}\Sigma)$ for all $A, B \in \Sigma$.

Corollary 3. [1] *The class of quasigroups isotopic to groups is characterized by the following identity:*

$$x(y\backslash((z/u)v)) = ((x(y\backslash z))/u)v.$$

Theorem 4. *The invertible algebra $(Q; \Sigma)$ is a principally isotopic to a group if and only if the following second-order formula:*

$$A({}^{-1}A(B(x, z), y), A^{-1}(u, B(w, y))) =$$

$$= A({}^{-1}A(B(w, z), y), A^{-1}(u, B(x, y))).$$

is valid in the algebra $(Q; \Sigma \cup \Sigma^{-1} \cup {}^{-1}\Sigma)$ for all $A, B \in \Sigma$.

Corollary 5. *The class of quasigroups isotopic to abelian groups is characterized by the following identity:*

$$((xz)/y)(u\backslash(wy)) = ((wz)/y)(u\backslash(xy)).$$

[1] V. D. Belousov, *Globaly associative systems of quasigroups*, ***Matematicheskii Sbornik***, vol. 55(97) (1961), no. 2, pp. 221–236.