

- THOMAS G. KUCERA, MARCOS MAZARI-ARMIDA, *On universal modules with pure embeddings*.

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This paper arose out of the realization of the second author that some notions of the theory of abstract elementary classes can be used to generalize a result of Shelah [?, 1.2] concerning the existence of universal reduced torsion-free abelian groups with respect to pure embeddings. The contribution of the first author was limited to helping him expand and extend the results to theories of modules.

We show that certain classes of modules have universal models with respect to pure embeddings.

THEOREM 1. *Let R be a ring, T a first-order theory with an infinite model extending the theory of R -modules and $\mathbf{K}^T = (\text{Mod}(T), \leq_{pp})$ (where \leq_{pp} stands for pure submodule). Assume \mathbf{K}^T has joint embedding and amalgamation.*

If $\lambda^{|T|} = \lambda$ or $\forall \mu < \lambda (\mu^{|T|} < \lambda)$, then \mathbf{K}^T has a universal model of cardinality λ .

We begin the study of limit models for classes of R -modules with joint embedding and amalgamation. As a by-product of this study, we characterize limit models of countable cofinality in the class of torsion-free abelian groups with pure embeddings, answering Question 4.25 of [?].

THEOREM 2. *If G is a (λ, ω) -limit model in the class of torsion-free abelian groups with pure embeddings, then $G \cong \mathbb{Q}^{(\lambda)} \oplus \prod_p \overline{\mathbb{Z}}_{(p)}^{(\lambda)}^{(\aleph_0)}$.*

[Maz]MARCOS MAZARI-ARMIDA, *Algebraic description of limit models in classes of abelian groups*, **preprint**, <https://arxiv.org/abs/1810.02203>.

[Sh17]SAHARON SHELAH, *Universal Structures*, **Notre Dame Journal of Formal Logic**, vol. 58 (2017), no. 2, pp. 159–177.