▶ DAMIR ZAINETDINOV, *Limitwise monotonic reducibility of sets and Σ-definability of abelian groups*.

N.I. Lobachevsky Institute of Mathematics and Mechanics, Kazan Federal University, 18 Kremlyovskaya St., Kazan, Russian Federation.
E-mail: `damir.zh@mail.ru`.

In my talk I will consider limitwise monotonic reducibility (lm-reducibility for short) of sets via Σ-definability of abelian groups. The notion of lm-reducibility of sets via the limitwise monotonic operator was introduced in [1]. The main results obtained with the investigation of limitwise monotonic functions, sets, and sequences can be found in papers [2, 3].

DEFINITION. Let sets $A, B \subseteq \mathbb{N}$. We define the following family of initial segments:

$$\mathcal{F}(A) = \{\mathbb{N} \upharpoonright n : n \in A\}.$$

Then $A \leq_{lm} B \iff \mathcal{F}(A) \sqsubseteq_{\Sigma} \mathcal{F}(B)$, where definition of Σ-reducibility on the families can be found in [4].

We consider an abelian group $G(A)$ in the following form:

$$G(A) = \bigoplus_{n \in A} \left(\bigoplus_{m \in \mathbb{N}} \mathbb{Z}_{p^n} \right),$$

where $\mathbb{Z}_{p^n}$ – cyclic group of order p^n and p is prime.

The main result of my talk is to obtain a description of the lm-reducibility of sets on the language of Σ-definability of abelian groups.

THEOREM 1. *The family $\mathcal{F}(A)$ is Σ-definable in the hereditarily finite superstructure $\mathbb{HF}(G(A))$ over the group $G(A)$.*

THEOREM 2. *Let $A, B \subseteq \mathbb{N}$. Let $G(A)$ and $G(B)$ be abelian groups defined for sets A and B, respectively. Then $A \leq_{lm} B$, if and only if the group $G(A)$ is Σ-definable in the hereditarily finite superstructure $\mathbb{HF}(G(B))$.*

[1] FAIZRAHMANOV M., KALIMULLIN I., ZAINETDINOV D., *Maximality and minimality under limitwise monotonic reducibility*, **Lobachevskii Journal of Mathematics**, vol. 35 (2014), no. 4, pp. 333–338.

[2] DOWNEY R., KACH A., TURETSKY D., *Limitwise monotonic functions and their applications*, **Proceedings of the 11th Asian Logic Conference. World Scientific**, (2011), pp. 59–85.

[3] KALIMULLIN I., KHOUSSAINOV B., MELNIKOV A., *Limitwise monotonic sequences and degree spectra of structures*, **Proceedings of the American Mathematical Society**, vol. 141 (2013), no. 9, pp. 3275–3289.

[4] KALIMULLIN I., PUZARENKO V., *Reducibility on families*, **Algebra and Logic**, vol. 48 (2009), no. 1, pp. 20–32.