

- NIKOLAY BAZHENOV, MANAT MUSTAFA, AND MARS YAMALEEV, *Computable reducibility, and isomorphisms of distributive lattices*.

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A standard tool for classifying computability-theoretic complexity of equivalence relations is provided by computable reducibility. Let  $E$  and  $F$  be equivalence relations on  $\omega$ . The relation  $E$  is *computably reducible* to  $F$ , denoted by  $E \leq_c F$ , if there is a total computable function  $f(x)$  such that for all  $x, y \in \omega$ ,

$$(x E y) \Leftrightarrow (f(x) F f(y)).$$

The systematic study of computable reducibility was initiated by Ershov [1, 2].

Let  $\alpha$  be a computable non-zero ordinal. An equivalence relation  $R$  is  $\Sigma_\alpha^0$  *complete* (for computable reducibility) if  $R \in \Sigma_\alpha^0$  and for any  $\Sigma_\alpha^0$  equivalence relation  $E$ , we have  $E \leq_c R$ . The article [3] provides many examples of  $\Sigma_n^0$  complete equivalence relations, which arise in a natural way in recursion theory. In [4], it was proved that for each of the following classes  $K$ , the relation of computable isomorphism for computable members of  $K$  is  $\Sigma_3^0$  complete: trees, equivalence structures, and Boolean algebras.

We prove that for any computable successor ordinal  $\alpha$ , the relation of  $\Delta_\alpha^0$  isomorphism for computable distributive lattices is  $\Sigma_{\alpha+2}^0$  complete. We obtain similar results for Heyting algebras, undirected graphs, and uniformly discrete metric spaces.

[1] YU. L. ERSHOV, *Positive equivalences*, **Algebra and Logic**, vol. 10 (1971), no. 6, pp. 378–394.

[2] YU. L. ERSHOV, *Theory of numberings* (in Russian), Nauka, Moscow, 1977.

[3] E. IANOVSKI, R. MILLER, K. M. NG, AND A. NIES, *Complexity of equivalence relations and preorders from computability theory*, **Journal of Symbolic Logic**, vol. 79 (2014), no. 3, pp. 859–881.

[4] E. FOKINA, S.-D. FRIEDMAN, AND A. NIES, *Equivalence relations that are  $\Sigma_3^0$  complete for computable reducibility*, **Logic, Language, Information and Computation** (Luke Ong and Ruy de Queiroz, editors), Lecture Notes in Computer Science, vol. 7456, Springer, Berlin, 2012, pp. 26–33.