▸ ANDREY MOROZOV, JAMALBEK TUSSUPOV, *On minimal elements in the* Δ–*reducibility on families of predicates.*
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Fix some countable set U. By *predicate* here we mean an arbitrary subset of an arbitrary finite Cartesian power of U. We study two kinds of reducibilities on finite families of predicates.

We say that a predicate R is Δ–*definable over the predicates* $P_1, \dots , P_k$ if R itself and its complement can be defined in the structure $\langle U; P_1, \dots , P_k\rangle$ by means of $\exists$–formulas with parameters.

Let $S_0 = \{P_0, \dots , P_{k-1}\}$ and S_1 be two finite families of predicates. We say that S_0 *is* Δ–*definable in* S_1, if all the predicates in S_0 are Δ–definable in S_1 and we denote this fact as $S_0 \leqslant^0_\Delta S_1$. If $S_0 \leqslant^0_\Delta S_1$ and $S_1 \leqslant^0_\Delta S_0$ then we denote this fact as $S_0 \equiv^0_\Delta S_1$. The relation $\leqslant^0_\Delta$ is a preordering, $\equiv^0_\Delta$ is an equivalence and the quotient $\leqslant^0_\Delta/_{\equiv^0_\Delta}$ defines an upper semilattice in which the least upper bound of elements $S_0/_{\equiv^0_\Delta}$ and $S_1/_{\equiv^0_\Delta}$ equals to $(S_0 \cup S_1)/_{\equiv^0_\Delta}$ and $\perp^0_\Delta = \varnothing/_{\equiv^0_\Delta}$ is the smallest element. Denote this semilattice by D^0_Δ.

If we consider families of predicates up to isomorphism, we arrive at the notion of Δ–*reducibility on families of predicates*. We say that a finite family of predicates S_0 Δ–*reduces to a finite family* S_1 (and denote this as $S_0 \leqslant_\Delta S_1$), if there exists a finite family of predicates S' such that $S'_0 \leqslant^0_\Delta S_1$ and S'_0 is a conjugate of S_0 by means of some permutation on U.

If $S_0 \leqslant_\Delta S_1$ and $S_1 \leqslant_\Delta S_0$ then we denote this fact as $S_0 \equiv_\Delta S_1$. The quotient $\leqslant_\Delta/_{\equiv_\Delta}$ defines a structure D_Δ, which is a partial order with smallest element $\perp_\Delta = \varnothing/_{\equiv_\Delta}$.

Theorem 1

1. The structure D_Δ fails to be an upper semilattice.
2. The families consisting of unary predicates define in D_Δ an ideal of order type ω.

Theorem 2 Each of the structures $D^0_\Delta \setminus \{\perp^0_\Delta\}$ and $D_\Delta \setminus \{\perp_\Delta\}$ contains 2^ω minimal elements.

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