

- ▶ ANGUS MACINTYRE, *Model theory of adèles. Arithmetic equivalence.*

Queen Mary University of London, UK.

E-mail: a.macintyre@qmul.ac.uk.

Let A_K be the ring of adèles of a number field K . Only after Ax had given his analyses of uniform definability and decidability for the completions of K at its standard absolute values (fifty years ago) could one give informative analyses of the definability and decidability for the individual A_K . This was first done, early on, by Weisspfenning. Much later Derakhshan and I have given a more algebraic treatment purely in the language of rings. Still, many questions remain unanswered, notably that of definability and decidability uniformly in K . This is related to basic issues of unbounded ramification (going back to Herbrand's work in algebraic number theory). Some of these issues will be sketched, but the main emphasis will be on a question posed in other terms by number theorists more than eighty years ago. The question asks to what extent A_K determines K . It has been known for a long time that A_K does not determine K (up to isomorphism) in general, and much fine structure has been discovered (involving Galois theory, zeta functions, class numbers, etc). In the talk I will give a thorough analysis of elementary equivalence for adèle rings, and show that it coincides with isomorphism. I also reformulate some work of the number theorists to show that for any K there are at most finitely many L so that A_K and A_L are isomorphic.

This work is joint with J.Derakhshan (Oxford).