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Approximately computable equivalence structures

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In the past, we investigated computable, computably enumerable, and co-computably enumerable equivalence structures and their isomorphisms [2, 3]. In recent years, various authors investigated approximate computability for sets and reducibilities. We introduce and study the notions of generic and coarse computability for equivalence structures and their isomorphisms [1]. A binary relation R on ω is *generically computable* if there is a partial computable function $\varphi : \omega^2 \to \{0, 1\}$ such that on its domain, φ coincides with the characteristic function of R and, furthermore, φ is defined on $A \times A$ for a computably enumerable set A of asymptotic density 1. A set $B \subseteq \omega$ is called *R-faithful* if, whenever aRb, then $a \in B$ iff $b \in B$. We say that a generically computable R is faithfully generically computable if the corresponding set A is R-faithful. We show that every equivalence structure has a generically computable copy. We also show that an equivalence structure $\mathcal{E}$ has a faithfully generically computable copy if and only if $\mathcal{E}$ has an infinite faithful substructure with a computable copy.

An equivalence structure $\mathcal{E} = (\omega, E)$ is *coarsely computable* if there is a computable binary relation C such that E and C agree on a set $A \subseteq \omega$ of asymptotic density 1. The structure $\mathcal{E}$ is faithfully coarsely computable if A is both C-faithful and E-faithful. Every equivalence structure has a coarsely computable copy. Not every faithfully coarsely computable equivalence structure has a faithfully generically computable copy, and not every equivalence structure has a faithfully coarsely computable copy. We also investigate generically and coarsely computable isomorphisms and how their categoricity differs from computable categoricity.

[1] W. Calvert, D. Cenzer, and V. Harizanov, Generically computable equivalence structures and isomorphisms, https://arxiv.org/abs/1808.02782

[2] W. Calvert, D. Cenzer, V. Harizanov and A. Morozov, Effective categoricity of equivalence structures, **Annals of Pure and Applied**

Logic 141 (2006), pp. 61–78.

[3] D. Cenzer, V. Harizanov, and J. Remmel, Σ^0_1 and Π^0_1 equivalence structures, **Annals of Pure and Applied Logic** 162 (2011), pp. 490–503.