

- LAURENT BIENVENU, BARBARA F. CSIMA, AND MATTHEW HARRISON-TRAINOR, *Some questions of uniformity in algorithmic randomness*. LaBRI, CNRS & Université de Bordeaux, 351 Cours de la Libération, 33405 Talence, France.  
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The  $\Omega$  numbers—the halting probabilities of universal prefix-free machines—are known to be exactly the Martin-Löf random left-c.e. reals [3, 4, 5]. It was previously open however whether this equivalence was uniform, i.e., whether one can uniformly produce, from a Martin-Löf random left-c.e. real  $\alpha$ , a universal machine  $U$  whose halting probability is  $\alpha$  (see for example [1]). We answer this question in the negative. We also answer a question of Barmalias and Lewis-Pye [2] by showing that given a left-c.e. real  $\alpha$ , one cannot uniformly produce a left-c.e. real  $\beta$  such that  $\alpha - \beta$  is neither left-c.e. nor right-c.e.

[1] GEORGE BARMALIAS, *Aspects of Chaitin's Omega*, **Algorithmic Randomness: Progress and Prospects** (Johanna Franklin and Christopher Porter, editors), Springer, 2018, pp. 623–632.

[2] GEORGE BARMALIAS AND ANDREW LEWIS-PYE, *A note on the differences of computably enumerable reals*, **Computability and Complexity, Lecture Notes in Computer Science** vol. 10010, Springer, 2017, pp. 623–632.

[3] CRISTIAN S. CALUDE, PETER H. HERTLING, BAKHADYR KHOUSSAINOV, AND YONGGE WANG, *Recursively enumerable reals and Chaitin  $\Omega$  numbers*, **Theoretical Computer Science**, vol. 255 (2001), no. 1-2, pp. 125–149.

[4] GREGORY J. CHAITIN, *A theory of program size formally identical to information theory*, **Journal of the ACM**, vol. 22 (1975), pp. 329–340.

[5] ANTONIN KUČERA AND TED SLAMAN, *Randomness and recursive enumerability*, **SIAM Journal on Computing**, vol. 31 (2001), pp. 199–211.