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Proof complexity of quantified Boolean formulas (QBF) studies different formal calculi for proving QBFs and compares them with respect to the size of proofs. There exists a number of conceptually quite different QBF resolution calculi, modelling QBF solving approaches, as well as QBF cutting planes, algebraic systems, Frege systems, and sequent calculi. We give an overview of the relative proof complexity landscape of these systems.

From a complexity perspective it is particularly interesting to understand which lower bound techniques are applicable in QBF proof complexity. While some propositional techniques, such as feasible interpolation [3] and game-theoretic approaches [4], can be lifted to QBF, QBF proof complexity also offers completely different approaches that do not have analogues in the propositional domain. These build on strategy extraction, whereby from a refutation of a false QBF a countermodel can be efficiently constructed. Extracting strategies in restricted computational models (such as bounded-depth circuits) and exhibiting false QBFs where countermodels are hard to compute in the same computational model leads to lower bounds for the size of proofs in QBF calculi.

We explain this paradigm for prominent QBFs [2, 1]. For QBF Frege systems this approach even characterises QBF Frege lower bounds by circuit lower bounds [5]. This provides a strong link between circuit complexity and QBF proof complexity, unparalleled in propositional proof complexity.

This line of research also intrinsically connects to QBF solving as different QBF resolution calculi form the basis for different approaches in QBF solving such as QCDCL [7] and QBF expansion [6]. Thus QBF proof complexity provides the main theoretical tool towards an understanding of the relative power and limitations of these powerful algorithms.

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