NEIL THAPEN, *Induction, search problems and approximate counting.*
Institute of Mathematics, Czech Academy of Sciences, Zitná 25, 115 67 Praha 1, Czech Republic.
E-mail: thapen@math.cas.cz.

An important open problem in bounded arithmetic is to show that (in the presence of an oracle predicate) theories with more induction are strictly stronger when it comes to proving sentences of some fixed complexity. In classical fragments of Peano arithmetic, the $\Pi_1$ consequences of theories can be separated by consistency statements, and the $\Pi_2$ consequences by the growth-rate of definable functions. In bounded arithmetic, neither of these seems to be possible.

I will discuss this problem, and describe some recent progress on it. A particular instance of the problem is to find a $\forall \Sigma^b_1$ sentence which is provable in full bounded arithmetic but not in $T^2_2$ (that is, with induction restricted to $\Sigma^b_2$ formulas). In [1] we study the theory APC$_2$, which allows approximate counting of $\Sigma^b_1$ sets, and appears to have a broadly similar level of strength to $T^2_2$. We find such a $\forall \Sigma^b_1$ sentence separating APC$_2$ from full bounded arithmetic, using a probabilistic oracle construction based on a simplified switching lemma.