

- RUMEN DIMITROV, VALENTINA HARIZANOV, ANDREY MOROZOV, PAUL SHAFER, ALEXANDRA SOSKOVA, AND STEFAN VATEV, *Cohesive powers of ω* .
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A *cohesive power* of a computable structure is an effective analog of an ultrapower of the structure in which a cohesive set plays the role of an ultrafilter. We study the cohesive powers of computable copies of the structure $(\omega, <)$, i.e., the natural numbers with their usual order. By a *computable copy of $(\omega, <)$* , we mean a computable linear order $\mathcal{L} = (L, <)$ that is isomorphic to $(\omega, <)$, but not necessarily by a computable isomorphism. That is, the successor function of \mathcal{L} may not be computable. Our main findings are the following. First, recall that ζ denotes the order type of the integers, that η denotes the order type of the rationals, and that $\omega + (\eta \times \zeta)$ (often also written $\omega + \zeta\eta$) is familiar as the order type of countable non-standard models of Peano arithmetic.

1. If \mathcal{L} is a computable copy of $(\omega, <)$ with a computable successor function, then every cohesive power of \mathcal{L} has order type $\omega + (\eta \times \zeta)$.
2. There is a computable copy \mathcal{L} of $(\omega, <)$ with a **non**-computable successor function such that every cohesive power of \mathcal{L} has order type $\omega + (\eta \times \zeta)$.
3. Most interestingly, there is a computable copy \mathcal{L} of $(\omega, <)$ (with a necessarily non-computable successor function) having a cohesive power that is **not** of order type $\omega + (\eta \times \zeta)$.