

- RUMEN DIMITROV, VALENTINA HARIZANOV, ANDREY MOROZOV, PAUL SHAFER, ALEXANDRA SOSKOVA, AND STEFAN VATEV, *Cohesive powers of  $\omega$* .  
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A *cohesive power* of a computable structure is an effective analog of an ultrapower of the structure in which a cohesive set plays the role of an ultrafilter. We study the cohesive powers of computable copies of the structure  $(\omega, <)$ , i.e., the natural numbers with their usual order. By a *computable copy* of  $(\omega, <)$ , we mean a computable linear order  $\mathcal{L} = (L, <)$  that is isomorphic to  $(\omega, <)$ , but not necessarily by a computable isomorphism. That is, the successor function of  $\mathcal{L}$  may not be computable. Our main findings are the following. First, recall that  $\zeta$  denotes the order type of the integers, that  $\eta$  denotes the order type of the rationals, and that  $\omega + (\eta \times \zeta)$  (often also written  $\omega + \zeta\eta$ ) is familiar as the order type of countable non-standard models of Peano arithmetic.

1. If  $\mathcal{L}$  is a computable copy of  $(\omega, <)$  with a computable successor function, then every cohesive power of  $\mathcal{L}$  has order type  $\omega + (\eta \times \zeta)$ .
2. There is a computable copy  $\mathcal{L}$  of  $(\omega, <)$  with a **non**-computable successor function such that every cohesive power of  $\mathcal{L}$  has order type  $\omega + (\eta \times \zeta)$ .
3. Most interestingly, there is a computable copy  $\mathcal{L}$  of  $(\omega, <)$  (with a necessarily non-computable successor function) having a cohesive power that is **not** of order type  $\omega + (\eta \times \zeta)$ .