

- OSVALDO GUZMAN, *The ultrafilter and almost disjointness numbers.*

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The *cardinal invariants of the continuum* are certain uncountable cardinals that are less or equal to the cardinality of the real numbers. This relation and nonrelation between these cardinals has been deeply studied by set theorists. In this talk, we will focus on the following two invariants: The *ultrafilter number* \mathfrak{u} , which is defined as the smallest size of a base of an ultrafilter, and the *almost disjointness number* \mathfrak{a} , which is the smallest size of a MAD family. The consistency of the inequality $\mathfrak{a} < \mathfrak{u}$ is well known and easy to prove. The consistency of the inequality $\mathfrak{u} < \mathfrak{a}$ is much harder to obtain. It was Shelah who proved that, under the assumption that there is a measurable cardinal, there is a model of $\omega_1 < \mathfrak{u} < \mathfrak{a}$. In spite of the beauty of the result, the following questions remained open:

(Shelah) Does $CON(ZFC)$ implies $CON(ZFC + \mathfrak{u} < \mathfrak{a})$?

(Brendle) Is it consistent that $\omega_1 = \mathfrak{u} < \mathfrak{a}$?

In this talk, we are going to see how to provide a positive answer to both questions. This is joint work with Damjan Kalajdzievski. No previous knowledge of cardinal invariants of the continuum is needed for the talk.