▸ ULRICH KOHLENBACH, *Local proof-theoretic foundations, proof-theoretic tameness and proof mining.*
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Recently, John Baldwin pointed to a 'paradigm shift in model theory' stressing that while early 20th century logic focused on the formalization of all of mathematics, model theory increasingly studied specific areas of mathematics (local formalizations) with an emphasis on tame structures ([1]). We will argue that also the successful use of proof-theoretic methods in core mathematics ('proof mining', [2]) in recent decades was made possible by developing logical metatheorems tailored for applications to particular classes of theorems and proofs in specific areas of mathematics. In analysis, these classes of theorems (e.g. convergence statements), however, do involve arithmetic (together with analytical and geometric structures) and so are not tame in the model-theoretic sense but could in principle display Gödelian or huge growth phenomena. It is an empirical fact, though, that with a few notable exceptions (which still are primitive recursive in the sense of Gödel's T), proofs in existing ordinary analysis are largely tame in the sense of allowing for the extraction of bounds of rather low complexity. To determine the amount of 'proof-theoretic tameness' in a given proof requires a proof-theoretic analysis in each case. We will discuss two recent applications of proof mining, one of which displays a highly tame (polynomial) behavior ([3]) whereas the other one as it stands uses primitive recursion of type-1 level ([4]).

[1] Baldwin, J.T., Model theory and the philosophy of mathematical practice. Formalization without foundationalism. xi+352pp., Cambridge University Press, 2018.

[2] Kohlenbach, U., Applied Proof Theory: Proof Interpretations and their Use in Mathematics. Springer Monographs in Mathematics. xx+536pp., Springer Heidelberg-Berlin, 2008.

[3] Kohlenbach, U., A polynomial rate of asymptotic regularity for compositions of projections in Hilbert space. Foundations of Computational Mathematics **19**, pp. 83-99 (2019).

[4] Kohlenbach, U., Sipoş, A., The finitary content of sunny nonexpansive retractions. arXiv:1812.04940. Submitted.