▸ BARTOSZ WCISŁO, *Topological models of arithmetic.*
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Our talk concerns the following question: which topological spaces X can be equipped with continuous operations $S : X \to X$, $+ : X^2 \to X$, and $\times : X^2 \to X$ such that $(X, +, \times, S)$ becomes a model of Peano Arithmetic (PA)?

By a result of Malitz, Mycielski, Reinhardt and (independently) Friedman, every theory in a countable signature has a model X which is a Polish space such that all definable relations are Borel (in fact, $F_\sigma \cap G_\delta$). Of course, there are a number of theories in functional signatures which have interesting topological models, i.e., models in which operations are continuous. Prominent examples include topological groups and rings where X can be chosen to be a particularly well-behaved space, for instance a manifold.

Ali Enayat asked whether there exists a Polish space X such that PA has a topological model $(X, +, \times, S)$. In joint work with Ali Enayat and Joel David Hamkins, we have obtained some partial results concerning this question. In particular, we know that any extension of PA has a topological model whose underlying space are rational numbers $\mathbb{Q}$. We have also shown that no finite-dimensional manifold or a compact Hausdorff space can be a model of PA.

If time allows, we will also present some additional results linking topology to the arithmetical structure of the model. In particular, it can be shown that in every cardinality there are topological spaces X which can be endowed with continuous operations making them models of PA such that not every model of PA (in the same cardinality) can be obtained as a topological model with the underlying space X.

All results in the talk are a joint work of Ali Enayat, Joel David Hamkins, and the author.

[1] Ali Enayat, Joel David Hamkins, Bartosz Wcisło, *Topological Models of Arithmetic*, preprint available at https://arxiv.org/abs/1808.01270.

[2] Jerome Malitz, Jan Mycielski, Willian Reinhardt, *The Axiom of Choice, the Löwenheim–Skolem Theorem and Borel Models*, ***Fundamenta Mathematicae,*** , vol. 137 (1991), no. 1, pp.53–58.

[3] Jerome Malitz, Jan Mycielski, Willian Reinhardt, *Erratum to the paper: The Axiom of Choice, the Löwenheim–Skolem Theorem and Borel Models*, ***Fundamenta Mathematicae,*** , vol. 140 (1992), no. 2, p. 197.