▶ NIKOLAY BAZHENOV, HRISTO GANCHEV, AND STEFAN VATEV, *Computable embeddings for pairs of linear orderings.*
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Friedman and Stanley [3] introduced the notion of *Borel embedding* to compare complexity of the classification problems for classes of countable structures. Calvert, Cummins, Knight, and Miller [1] (see also [2] and [4]) developed two notions, *computable embeddings* and *Turing computable embeddings*, as effective counterparts of Borel embeddings.

We follow the approach of [1] and study computable embeddings for pairs of structures, i.e. for classes $\mathcal{K}$ containing precisely two non-isomorphic structures. Our motivation for investigating pairs of structures is two-fold. These pairs play an important role in computable structure theory and also they constitute the simplest case, which is significantly different from the case of one-element classes. It is not hard to show that for any computable structures $\mathcal{A}$ and $\mathcal{B}$, the one-element classes $\{\mathcal{A}\}$ and $\{\mathcal{B}\}$ are equivalent with respect to computable embeddings. On the other hand, computable embeddings induce a non-trivial degree structure for two-element classes consisting of computable structures.

In this talk we will concentrate on the pair of linear orders ω and $\omega^\star$. By $\deg_{tc}(\{\omega, \omega^\star\})$ we denote the degree of the class $\{\omega, \omega^\star\}$ under Turing computable embeddings. Quite unexpectedly, it turns out that a seemingly simple problem of studying computable embeddings for classes from $\deg_{tc}(\{\omega, \omega^\star\})$ requires developing new techniques.

We give a necessary and sufficient condition for a pair of structures $\{\mathcal{A}, \mathcal{B}\}$ to belong to $\deg_{tc}(\{\omega, \omega^\star\})$. We also show that the pair $\{1+\eta, \eta+1\}$ is the greatest element inside $\deg_{tc}(\{\omega, \omega^\star\})$, with respect to computable embeddings. More interestingly, we prove that inside $\deg_{tc}(\{\omega, \omega^\star\})$, there is an infinite chain of degrees induced by computable embeddings.

[1] W. Calvert, D. Cummins, J. F. Knight, S. Miller, *Comparing classes of finite structures*, ***Algebra Logic***, vol. 43, (2004), no. 6, pp. 374–392.

[2] J. Chisholm, J. F. Knight, S. Miller, *Computable embeddings and strongly minimal theories*, ***Journal of Symbolic Logic***, vol. 72 (2007), no. 3, pp. 1031–1040.

[3] H. Friedman, L. Stanley, *A Borel reducibility theory for classes of countable structures*, ***Journal of Symbolic Logic*** vol. 54 (1989), no. 3, pp. 894–914.

[4] J. F. Knight, S. Miller, M. Vanden Boom, *Turing computable embeddings*, ***Journal of Symbolic Logic***, vol. 72 (2007), no. 3, pp. 901–918.