ANAHIT CHUBARYAN, GARIK PETROSYAN, SERGEY SAYADYAN. Monotonous and strong monotonous properties of some propositional proof systems for Classical and Non Classical Logics.
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For some propositional proof system of classical and non-classical logics we investigate the relations between the lines ($t$-complexities) and sizes ($l$-complexities) of proofs for minimal tautologies, which are not a substitution of a shorter tautology of this logic, and results of a substitutions in them. For every minimal tautology $\varphi$ of fixed logic by $S(\varphi)$ is denoted the set of all tautologies, which are results a substitutions in $\varphi$.

**Definition 1.** The proof system $\Phi$ is called $t$-monotonous ($l$-monotonous), if for every minimal tautology $\varphi$ of this system and for every formula $\psi$ from $S(\varphi)$ $t^\Phi(\varphi) \leq t^\Phi(\psi)$ ($l^\Phi(\varphi) \leq l^\Phi(\psi)$).

**Definition 2.** The proof system $\Phi$ is called $t$-strong monotonous ($l$-strong monotonous), if for every non-minimal tautology $\psi$ of this system there is such minimal tautology $\varphi$ of this system such that $\psi$ belong to $S(\varphi)$ and $t^\Phi(\psi) \leq t^\Phi(\varphi)$ ($l^\Phi(\psi) \leq l^\Phi(\varphi)$).

Formerly it is proved in [1], that Frege systems for classical and non-classical logics are neither $t$-monotonous nor $l$-monotonous.

Now we consider the following systems: propositional resolution systems $RC, RI, RJ$ for classical, intuitionistic and Johanssons logics accordingly, eliminations systems $E, EI, EJ$, based on the determinative normal forms for the same logics [2], and the system $GS$, based on generalization of splitting method [3].

**Theorem 1.** The systems $RC, RI$ and $RJ$ are $t$-strong monotonous ($l$-strong monotonous), but neither of them is $t$-monotonous ($l$-monotonous).

**Theorem 2.** Each of the systems $EC, EI, EJ$ and $GS$ is neither $t$-monotonous ($l$-monotonous) nor $t$-strong monotonous ($l$-strong monotonous).

**Acknowledgments** This work was supported by the RA MES State Committee of Science, in the frames of the research project Nr. 18T-1B034.

