

► DAVID PIERCE, *Apollonian proof*.

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This is about a method of proof that seems to have been passed over. Descartes suspected ancient geometers of using algebra, but covering their tracks. However, in the centuries since *La Géométrie* of 1637 [2], persons such as Wallis [7], de Witt [3], and Hamilton [5] who have reworked a basic result of Apollonius [1, I.49–50] have abandoned his visual proof.

Apollonius gives three ways to characterize a conic section: (i) an equation, involving a latus rectum, that we can express in Cartesian form; (ii) the proportion whereby the square on the ordinate varies as the abscissa or abscissas; (iii) an equation of a triangle with a parallelogram or trapezoid. Holding in an affine plane, the latter equation is not usefully translated into the lengths that Descartes has taught us to work with. With the equation, there is a proof-without-words of what today we consider a coordinate change. According to Rosenfeld [6], “Apollonius never mentions parabolic, elliptic, and hyperbolic turns, but no doubt that he used these transformations.” The better modern term for what Apollonius uses would be “scissors congruence” or “motivic measure” [4].

[1] APOLLONIUS OF PERGA, *Conics Books I–III*, Green Lion, 2000.

[2] RENÉ DESCARTES, *The Geometry of René Descartes*, Dover, 1954.

[3] Albert W. Grootendorst, editor. *Jan de Witt’s Elementa Curvarum Linearum, Liber Primus*, Springer, 2000.

[4] THOMAS C. HALES, *What is motivic measure?*, *Bulletin of the American Mathematical Society*, vol. 42 (2005), no. 2, pp. 119–135.

[5] HUGH HAMILTON, *A geometrical treatise of the conic sections, in which the properties of the sections are derived from the nature of the cone, in an easy manner, and by a new method*, J. Nourse, London, 1773.

[6] BORIS ROSENFELD, *The up-to-date complete scientific English translation of the great treatise of Apollonius of Perga with the most comprehensive commentaries*, www.personal.psu.edu/sxk37/Apollonius.html.

[7] JOHN WALLIS, *De Sectionibus Conicis, Nova Methodo Expositis, Tractatus*, Oxford, 1655.