▶ MARLENA FILA, PIOTR BŁASZCZYK, *Limits of diagrammatic reasoning.*
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We challenge theses of [3] and [4] concerning the Intermediate Value Theorem (IVT); we argue that a diagrammatic reasoning is reliable provided one finds a formula representing the diagram.

IVT states: If $(F, +, \cdot, 0, 1, <)$ is an ordered field, $f : [0, 1] \mapsto F$ is a continuous map such that $f(0)f(1) < 0$, then $f(x) = 0$, for some $x \in (0, 1)$. An accompanying diagram, diag(IVT), depicts a graph of f intersecting a line $(F, <)$, as the function values differ in sign.

(a) In [3], Brown argues that diag(IVT) guarantees the existence of an intersection point. (b) In [4], Giaquinto argues that diag(IVT) do not guarantee the existence thesis, since continuous functions include non-smooth functions that find no graphic representations.

(ad a) We show that IVT is equivalent to Dedekind Cuts principle (DC): If (A, B) is a Dedekind cut in $(F, <)$, then

$$(\exists! c \in F)(\forall x \in A)(\forall y \in B)[x \leq c \leq y].$$

We also provide a graphic representation for DC.

This equivalence justifies the claim that IVT is as obvious as DC. There is, however, no relation between diag(IVT) and diag(DC), all the more between diag(IVT) and the formula DC. Thus, Brown's claim has to be based on the analytic truth $IVT \Leftrightarrow DC$.

(ad b) Diagrams representing lines $(F, <)$ do not depict whether the field $(F, +, \cdot, 0, 1, <)$ is Euclidean (closed under the square root operation), or $(\mathbb{R}, +, \cdot, 0, 1, <)$, or a real-closed field; graphs of f do not distinguish between polynomial and smooth functions. IVT for polynomials, IVT_p, is valid in real-closed fields (these fields could be *bigger* or *smaller* than real numbers); in fact, IVT_p is the axiom for real-closed fields (next to the Euclidean condition).

Bolzano is believed to give the first proof of IVT. In fact, he sought to prove IVT_p, whilst IVT was just the lemma. Mislead by a diagram, Bolzano proved the theorem not as general as it could be: he proved only that IVT_p is valid in the domain of real numbers.

[1] Piotr Błaszczyk, *A purely algebraic Proof of the Fundamental Theorem of Algebra*, ***AUPC***, vol. 206 (2016), pp. 7–23.

[2] Bernard Bolzano, ***Rein analytischer Beweis***, Gotlieb Hasse, 1817.

[3] James R. Brown, *Proofs and Pictures*, ***Brit. J. Phil Sci.***, vol. 48 (1997), pp. 161–180.

[4] Marcus Giaquinto, *Crossing curves: A limit to the use of diagrams in proofs*, ***Philosophia Mathematica***, vol. 19 (2011), pp. 181–207.