▸ PIOTR BŁASZCZYK, *Axioms for Euclid's Elements book V, their consequences and some independence results.*
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Euclid's *Elements*, book V develops the theory of proportions as applied to magnitudes; it is the key theory for understanding Greek and early modern mathematics. By formalizing its definitions and the tacit assumptions behind its proofs, we reconstruct book V with its 25 propositions as an axiomatic theory.

The general term $\mu\varepsilon\gamma\varepsilon\theta o\varsigma$ covers line segments, triangles, convex polygones, circles, solids, angles, and arcs of circles. We formalize Euclid's magnitudes of the same kind (line segments being of one kind, triangles being of another, etc.) as an additive semigroup with a total order, $(M, +, <)$, characterized by the following five axioms:

E1 $(\forall x, y)(\exists n \in \mathbb{N})[nx > y]$,
E2 $(\forall x, y)(\exists z)[x < y \Rightarrow x + z = y]$,
E3 $(\forall x, y, z)[x < y \Rightarrow x + z < y + z]$,
E4 $(\forall x)(\forall n \in \mathbb{N})(\exists y)[ny = x]$,
E5 $(\forall x, y, z)(\exists v)[x : y :: z : v]$.

We show that E4 follows from E1-E3, E5; we prove the independence of the axioms E1, E2, E3. We discuss the use of E1 in the proposition V.8; we show that E1 does not follow from the Dedekind completeness axiom (although it does follow from the completeness axiom in an orderd group). We interpret Greek proportion in an Archimedean ordered field, and offer an algebraic interpretation of the axiom *The whole is greater than the part*.

We present schemes of Euclid's propositions; they consist of algebraic formulae representing sequences of (grammatical) sentences, signs representing phrases that occur in the Greek text and references to the axioms, definitions, and other propositions. We discuss under what assumptions these schemes could be turned into modern proofs. Finally, we present algebraic paraphrases of all 25 propositions of book V as derived from the axioms E1-E5.

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[3] Johan L. Heiberg, ***Euclidis Elementa***, Teubneri, 1883–1888.

[4] Ian Mueller, ***Philosophy of Mathematics and Deductive Structure in Euclid's Elements***, MIT Press, 2006.