▸ PHILIPPE BALBIANI, AND TINKO TINCHEV, *Computability of contact logics with measure.*
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Contact logics [1] are propositional logics interpreted over Boolean contact algebras [3]. They stem from the point-free approaches of geometry put forward by Whitehead. Their language $\mathcal{L}(\leq, C)$ includes Boolean terms representing regions. Let $\mathbf{X}$ be a set of variables. The set of Boolean terms (s, t, etc) over $\mathbf{X}$ being denoted $\mathbf{T}(\mathbf{X})$, the set $\mathbf{A}(\mathbf{X})$ of atomic formulas over $\mathbf{X}$ consists of all expressions of the form $s \leq t$ ("s is part-of t") and $C(s,t)$ ("s is in contact with t"). The set of all formulas (φ, ψ, etc) over $\mathbf{X}$ is the least set $\mathbf{F}(\mathbf{X})$ containing $\mathbf{A}(\mathbf{X})$ and such that for all $\varphi, \psi \in \mathbf{F}(\mathbf{X})$: $\bot \in \mathbf{F}(\mathbf{X})$, $\neg\varphi \in \mathbf{F}(\mathbf{X})$ and $(\varphi \vee \psi) \in \mathbf{F}(\mathbf{X})$. Of interest are, of course, the sets of all valid formulas determined by the various classes of Boolean contact algebras one may consider. See [1, 5] for detailed investigations.

The combination of topological and size information is a fundamental issue for multifarious applications of spatial reasoning [4]. It can be realized by considering Boolean contact algebras with measure, i.e. algebraic structures (A, C, μ) where (A, C) is a Boolean contact algebra and μ is a positive finite measure on A. Contact logics with measure are extensions of contact logics. Their language $\mathcal{L}(\leq, C, \leq_m)$ contains all additional atomic formulas of the form $s \leq_m t$ ("the size of s is less or equal than the size of t"). Of interest are, again, the sets of all valid formulas determined by the various classes of Boolean contact algebras with measure one may consider.

Using complexity results about linear programming [2], we show that the set of all valid formulas determined by the class of all Boolean contact algebras with measure is in **coNP**. Our proof relies on the equivalence between the satisfiability of a given formula φ and the consistency of an associated system $\mathcal{S}_\varphi$ of linear inequalities. It uses the following facts: the computation of $\mathcal{S}_\varphi$ from φ is possible in non-deterministic polynomial time; if a system of k linear inequalities with integer coefficients of length at most n has a non-negative solution then it has a non-negative solution with at most k positive entries of length in $\mathcal{O}(k.(n + \log k))$.

[1] Balbiani, P., T. Tinchev, and D. Vakarelov, 'Modal logics for region-based theories of space', *Fundamenta Informaticæ* **81** (2007) 29–82.

[2] Chvátal, V., *Linear Programming,* Freeman (1983).

[3] Dimov, G., and D. Vakarelov, 'Contact algebras and region-based theory of space: a proximity approach – I', *Fundamenta Informaticæ* **74** (2006) 209–249.

[4] Gerevini, A., and J. Renz, 'Combining topological and size information for spatial reasoning', *Artificial Intelligence* **137** (2002) 1–42.

[5] Kontchakov, R., I. Pratt-Hartmann, and M. Zakharyaschev, 'Spatial reasoning with *RCC*8 and connectedness constraints in Euclidean spaces', *Artificial Intelligence* **217** (2014) 43–75.