

- IRIS VAN DER GIESSEN, *Intuitionistic provability logic*.

Department of Philosophy and Religious Studies, Utrecht University, Janskerkhof 13, The Netherlands.

E-mail: i.vandergiesen@uu.nl.

The provability logic GL is obtained by adding Gödel-Löb's axiom $\Box(\Box A \rightarrow A) \rightarrow \Box A$ to a Hilbert calculus for classical modal logic K . Here, $\Box A$ reads as ' A is provable'. Intuitionistic provability logic is given by K restricted to intuitionistic tautologies together with Gödel-Löb's axiom. In sequent style, this is obtained by adding the modal rule

$$\frac{\Box\Gamma, \Gamma, \Box A \Rightarrow A}{\Pi, \Box\Gamma \Rightarrow \Box A} \text{ GLR}$$

to a sequent calculus for intuitionistic propositional logic.

We study two calculi for intuitionistic provability logic. One is the terminating version of the other. For both systems we prove the admissibility of the cut rule. One proof uses syntactic methods, the other model-theoretic ones.

One calculus that we study is GL3i. This is the intuitionistic propositional calculus G3ip together with GLR. We obtain cut-admissibility by applying a syntactic method developed by Valentini [1], using a third induction parameter, called *width*.

The other calculus that we consider is GL4i, which is obtained by adding GLR to the terminating system G4ip [2]. Termination of GL4i is based on a loop-preventing proof search adopted from results by Bílková [3]. Cut-admissibility is shown using a semantic strategy as in [4].

Using these results, we establish Craig interpolation for intuitionistic provability logic. One of our aims is to use the terminating calculus GL4i to prove uniform interpolation.

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[2] DYCKHOFF, R., *Contraction-Free Sequent Calculi for Intuitionistic Logic*, **Association for Symbolic Logic**, vol. 57 (1992), no. 3, pp. 795–807.

[3] BÍLKOVÁ, M., *Interpolation in Modal Logics*, PhD Thesis, UK Praha, 2006.

[4] AVRON, A., *On Modal Systems Having Arithmetical Interpretations*, **Journal of Symbolic Logic**, vol. 49 (1984), no. 3, pp. 935–942.